Time-Varying Subjective Bad Regime Risk: New Framework For Understanding Asset Prices

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April 14, 2026

Abstract

In most equilibrium asset pricing models, investors are assumed to know the dynamics of firm cash flows and economic fundamentals for sure. I develop a new regime-switching framework that relaxes this assumption without losing mathematical rigor or empirical tractability. In the model, investors know true cash flow dynamics conditional on macroeconomic or asset-level expansions and contractions, but do not know transition dynamics between the regimes. Using a new solution concept, I obtain asset prices in closed form for given subjective beliefs about future regime likelihood and show how beliefs can be jointly backed out from the term structures of long-term bond yields and individual and S&P500 VIX measures and their futures.

The new framework qualitatively explains a wedge between investor risk premium and observed average excess returns, a cyclicality gap between subjective risk premia and realized excess returns, a pro-cyclical gap between VIX and realized volatility, yield curve inversion/reversions without a regime shift, return predictability, narrative momentum, market crashes without macroeconomic contractions, and fast price recoveries during economic downturns.[1]

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[1]Calibration methodology and some implications are not fully exposed in this manuscript, but can be provided later upon request.

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1 Introduction

Most theoretical asset pricing models assume that at least up to a few parameters or states, investors know the true dynamics of firm cash flows and economic fundamentals. In disaster risk models, for instance, investors must know or think that disaster probability is mean-reverting (Tsai and Wachter 2015) or that it follows a regime-switching process (Ghaderi et al. 2022). Such modeling designs imply investors are certain about the fundamental nature of individual firms or macroeconomy and no news can affect the general properties of perceived risk dynamics. This paper posits that such stability in perceived dynamics makes existing models too rigid relative to the real world and many anomalies in asset pricing literature can be understood as a consequence of the variation in perceived dynamics of cash flows and fundamentals.

The instability of perceived macroeconomic dynamics should be obvious: changing world orders, wars, pandemics, environmental threats, and technological breakthroughs, to name the few, all affect the dynamics of fundamentals in ways humans cannot confidently anticipate. At the firm level, struggling startups occasionally rise to global dominance and dominant firms sometimes decline permanently. It is thus reasonable to assume that investors adjust their understanding of macroeconomic and firm-level risk dynamics over time and in response to major news.

I propose a new regime-switching model in which investors know the dynamics of asset cash flows and aggregate consumption conditional on macroeconomic and firm-level expansions and contractions, but do not know the transition dynamics between the two regimes. Instead, they make decisions using their subjective beliefs about regime likelihood over different future horizons. Regime transition dynamics are revealed ex-post but are not known or priced ex-ante. The model assumes knowing regime transition dynamics ex-ante is infeasible, because the risk landscape constantly evolves due to unprecedented developments. Hence, past observations are not enough to form a conviction that macroeconomic or idiosyncratic regime dynamics abide by a particular stochastic law. This is the key differentiating insight that drives novel implications.

Most regime-switching models require the specification of transition dynamics for model tractability and mathematical rigor. A major methodological novelty of this paper is developing a new solution concept that makes it possible to abstract away from regime dynamics and focus on subjective regime beliefs over different future horizons instead. Such design is feasible, because the solution concept facilitates model solution under arbitrary dynamically consistent regime beliefs without any assumptions about

their dynamics.[2] The essence of the solution concept is as follows: decompose any future horizon into finite segments of equal length and assume regimes are determined at the start point of each segment and persist for sure until its end. Under such decomposition, prices can be obtained in closed-form for arbitrary beliefs using backward induction and the law of iterated expectations. The actual model-implied prices are then limit cases of the prices under the finite decomposition, where the limit is taken as the segment lengths shrink to zero. In the limit, regimes can switch any time and persist for sure over infinitesimally small intervals only. Thus, the model can be solved without imposing any restrictions on perceived regime dynamics.

The framework models any asset as a portfolio of its systematic cash flow stream and its idiosyncratic cash flow stream. The price of any asset is thus the sum of the values of the two streams. Systematic cash flows are correlated with the state of the macroeconomy, while idiosyncratic cash flows are not. Each cash flow stream has two regimes. Regimes of systematic cash flows are macroeconomic and regimes of idiosyncratic cash flows are idiosyncratic. The price of each stream is a weighted average of the standard Gordon growth formulas that differ in discount rates and subjective growth expectations corresponding to different future disjoint time periods. The formulas corresponding to time periods closer to the present have strictly higher weight.

For each future time interval, the discount rates and growth expectations of each cash flow type are subjective-belief-weighted averages of the discount rates and growth expectations corresponding to two single-regime worlds: one with low-volatility growth and another with high-volatility decline. A good regime is a natural state of a growing firm or an economy, while a bad regime is a state in which one or more disruptions induce cash flow volatility and decline. Systematic regime beliefs is the only source of time variation in systematic cash flow growth expectations, while idiosyncratic regime beliefs is the only source of time variation in idiosyncratic cash flow growth expectations. Discount rates of each cash flow stream exhibit time variation due to changing systematic bad regime risk only. However, idiosyncratic regime beliefs still affect discount rates on total cash flows since they affect perceived exposure of the asset to macro risk. The discount rates and growth expectations thus differ across time and horizon and can respond instantly to abstract news such as the possibility of wars and pandemics, or of internal struggles within a firm.

Subjective beliefs about macroeconomic regimes can be backed out from S&P 500

[2]Dynamic consistency here is subjective: it merely requires that beliefs investors expect to have tomorrow about a future point s equal their beliefs about s today.

VIX term structure by matching model implied expected market volatilities over different horizons to S&P 500 VIX and its futures. Similarly, subjective beliefs about idiosyncratic regimes can be backed out from VIX term structure of S&P500 and individual stocks. Because VIX futures are not available at horizons beyond 5 years and the current monetary policy can have only informational effect on long-horizon real risk-free yields, long-horizon systematic regime beliefs can be backed out by matching model-implied risk-free yields to their empirical counterparts.

Because the framework of this paper does not rely on true regime transition dynamics, the concept of regimes in the paper is slightly but meaningfully different from that in the existing literature. In existing models (e.g. Berrada et al. (2018) and Andrade et al. (2023)), regimes are by definition persistent and have a particular expected duration. In this paper, a bad regime is simply a cash-flow declining episode. The market might expect it to be temporary or persistent, and time and horizon-varying beliefs about it are backed out from VIX and bond yield curve data.

VIX measures are model-free and aggregate many pieces of information about macroeconomy and individual firms that cannot be represented using primitive variables (e.g. political risks, pandemics, climate change etc). Hence, VIX is not just a method of calibrating a model. It is also a way of incorporating abstract sources of risk that matter for asset prices even if they do not always materialize. Variation due to such abstract news can explain many phenomena, including market crashes without macroeconomic contractions and fast market recoveries in bad times despite declining cash flows. In fact, Ghosh et al. (2017) estimate the market SDF non-parametrically and conclude that leading asset pricing models would benefit from SDF augmentation using a component that generates market crashes without macroeconomic contractions. The framework of this paper accomplishes this in an empirically tractable way with a carefully motivated microfoundation.[3]

In general, the time$-t$ price P_t^i of asset i can be compactly expressed as

$$P_t^i = \frac{\tilde{D}_t^i}{\delta_{\tilde{D},t}^i} + \frac{e_t^i}{\delta_{e,t}^i},$$

where $\tilde{D}_t^i$ $\left(e_t^i\right)$ is a current level of systematic (idiosyncratic) cash flows, and $\delta_{\tilde{D},t}^i$ $\left(\delta_{e,t}^i\right)$

[3]While the subjective SDF in this paper is a function of aggregate consumption growth only, the SDF expressed w.r.t the true data generating process will have additional component due to the imperfect subjective beliefs. This can be shown using a straightforward change of measure without specifying what the true data generating process is.

is a function of systematic (idiosyncratic) regime beliefs and increases any time there is an increase in systematic (idiosyncratic) bad regime likelihood about any future point of time. The framework nests virtually all results of standard continuous-time CRRA representative agent models once good regime subjective probabilities are uniformly set to equal one.

$\delta^i_{\tilde{D},t}$ and $\delta^i_{e,t}$ have parsimonious and fully calibrable closed-form expressions. Hence, the model can generate time series of prices that can be directly compared against data. This in turn implies the model need not be limited to explaining average asset pricing statistics and can be used to analyze specific time episodes or phenomena. This is challenging to do in other asset pricing models, since they depend on intractable time-varying quantities that cannot be reliably inferred from data (Campbell and Cochrane 1999; Gabaix 2014; Albuquerque et al. 2016; Seo and Wachter 2019), such as time-varying rare disaster probability.

Separating cash flows into systematic and idiosyncratic streams with their own regimes also facilitates cross-sectional analysis of asset prices and returns, a feature no other equilibrium regime-switching model shares, to the best of my knowledge. Incorporating idiosyncratic regimes in existing models is challenging, because specifying transition dynamics of idiosyncratic regimes is empirically infeasible. Even conceptually, it would require that investors have a fixed understanding of how individual firm outcomes evolve over time (e.g. they must think that idiosyncratic firm cash flows necessarily grow in the long run or necessarily stagnate/decline).

A notable technical result of the framework is a closed form expression for the individual asset return decomposition into a sum of the discount rate, cash flow growth surprise, risk free rate news, risk premium news, future systematic cash flow news and future idiosyncratic cash flow news. A striking feature of the decomposition is that although the subjective expected return always equals the discount rate, the same is not true about the observed average return. Because investors are not assumed to know regime transition dynamics, changing regime risk perceptions can drive a wedge between subjective expectations and average observed outcomes.

The rest of the paper is organized as follows. Section 1.1 provides a brief summary of novel implications, Section 2 describes the model, Section 3 derives closed-form model solutions, Section 4 discusses novel implications and Section 5 concludes by discussing several strands of follow-up work. Potential follow-up work includes the development of microfoundations for regime beliefs, applications to cryptocurrency pricing and investment management, and the relevance of the new framework for mapping investor

surveys to asset pricing models.

1.1 Summary of Novel Implications

1. Cyclicality Gap Between Subjective Risk Premia and Realized Excess Returns - Nagel and Xu (2023) show that the subjective risk premia measured by surveys exhibit much less cyclicality than realized excess returns. The model of this paper replicates the same pattern: the subjective expected returns always equal the risk premia (which exhibit modest counter-cyclicality), but realized returns reflect the effects of belief revision as well (which are unpredictable to the investor when forming expectations). Because most of the emerging threats have not materialized so far on average (as evidenced e.g. by VIX time series), periods of high risk are associated with subsequent favorable belief revisions and high average returns in the data. The pattern reflects uncertainty resolution ex-post and need not indicate investor irrationality if true regime dynamics are non-stationary and unknowable: past experiences have limited predictive power regarding a current disturbance if the latter is driven by unique circumstances the market has little experience of. In fact, Nagel and Xu (2023) also show high returns are not predictable out-of-sample at a time a forecast is made.

2. Growth Component of Average Returns - The model predicts the average excess returns of growing firms will be above the subjective risk premium of the investor because of gradually improving future outlook and compensation for unrealized risk. If "Googles" cannot be predicted in advance but are only revealed gradually ex-post, their average returns will reflect a drift due to gradually revising risk perceptions. Even if "Googles" can be predicted to some extent, their average returns will still reflect a positive drift on top of the subjective risk premium due to the gradual resolution of uncertainty in favor of successful outcomes. If along the way these firms experience lower incidence of bad regimes than the market anticipated, average stock returns will have additional positive component due to the unrealized risk. Both effects come on top of the distortions in subjective risk premia that might arise due to imperfect subjective beliefs.

   (a) The Growth component of returns would exhibit at the market level as well, since capitalization-weighted index outcomes are by construction tilted towards successful firms.

(b) If firm profitability is gradually revealed ex-post (i.e. a given outcome has predictive power only about short-horizon future outcomes), profitable firms will on average have positive revisions in idiosyncratic cash flow expectations, which could be behind the gross profitability anomaly of Novy-Marx (2013). In fact, Han et al. (2025) decompose firm profitability into systematic, industry-level and idiosyncratic components and show that the positive abnormal returns of profitable firms is exclusively driven by idiosyncratic ROA.

3. Return Predictability Linked to the Predictability of Fundamentals - The framework explains return predictability linked to the predictability of fundamentals as a consequence of changing risk perceptions. A few papers find that past earnings (profit trends) can predict future earnings (future profits) and abnormal returns that are not explained away by risk factors and occur in large part within the 3-day windows around earnings announcements (Bernard and Thomas 1990; Chan et al. 1996; Akbas et al. 2017; He and Narayanamoorthy 2020). The framework accommodates both phenomena under the assumption that the market is aware of the short-term auto-correlation in fundamentals. Because strong past earnings growth predicts strong earnings growth in the future (and vice versa), a good earnings news leads to downward revision in bad regime likelihood about the near term, which in turn contributes to positive returns. The abnormal returns will not be explained away by risk factors as long as the earnings news is in part driven by idiosyncratic cash flows, and a large part of the abnormal return will be realized within 3-day windows around earnings announcements as a result of uncertainty resolution regarding events that are informative about the future direction of cash flow growth. The latter point can perhaps be best understood using an analogy with prediction markets, in which investors can realize substantial gains after a correctly priced bet is realized even if the chance of it coming true was already high. In fact, the new framework is in line with the idea that the stock market implicitly embeds prediction markets around events that are informative about the future direction of cash flow growth.

4. The framework of this paper also provides a new insight regarding the Negative Relationship Between Past Earnings Growth and Future Long-Horizon Returns documented by Bordalo et al. (2019) and Bordalo et al. (2024). In the papers,

the authors explain the relationship by showing that analysts overextrapolate on past earnings, which leads to inflated prices and subsequently lower returns as expectations are disappointed. Although the framework of this paper can accommodate such mechanism[4], it also offers a new insight: as long as a series of good earnings news in the past decreases subjective future bad regime likelihood, risk premia corresponding to future horizons go down and result in lower realized returns all else equal. Although the mechanism is unsupported by data on analyst expectations, it could still play some role if analyst expectations are not a perfect proxy for the beliefs of the marginal investor.

5. Pro-cyclical Gap Between VIX and Realized Volatility - The model explains why expected volatility is higher than realized volatility in good times AND the opposite is true in bad times. In good times, expected volatility accounts for the possibility of a bad regime, but the latter has no effect on realized volatility unless a disruption arrives. In bad times, the expected volatility accounts for the possibility of a recovery, but the latter has no effect on realized volatility unless the recovery is in sight.

6. Risk-Free Yield Curve Inversion/Reversion Without a Regime Shift - The risk-free yield curve is determined by the term structure of regime beliefs irrespective of the state of the economy. If regime beliefs are incrementally pessimistic with increasing horizon, the yield curve will be downward-sloping. If regime beliefs are incrementally optimistic with increasing horizon, the yield curve will be upward-sloping. Hence, news that substantially increase subjective recession likelihood in the near term will invert the yield curve, and positive news in bad times that induce the market to expect recovery will revert the yield curve.

7. Market Crashes Without Macroeconomic Contractions and Fast Price Recoveries - Because $\delta^i_{\tilde{D},t}$ and $\delta^i_{e,t}$ terms are defined independently of cash flows and are functions of regime beliefs at different future horizons, market prices can react to information about potential firm-level or economy-wide disruptors even if they do not immediately affect cash flows or materialize at all. Since $\delta^i_{\tilde{D},t}$ and $\delta^i_{e,t}$ are backed out from VIX data and are strictly increasing functions of subjective

[4]Because model prices are obtained under arbitrary regime beliefs that subjectively satisfy dynamic consistency, the model neither a priori specifies nor a priori rules out any specific channel via which past earnings affect regime beliefs. Hence, the reduced-form model of diagnostic expectations can be directly combined with the equilibrium structure.

bad regime probabilities at different horizons, the model can replicate market crashes without macroeconomic contractions or price recoveries during economic downturns.

2 Model Description

2.1 Basic Setup

A representative investor with time-separable CRRA preferences prices all assets in continuous time. There is one risk-free asset and N risky assets. Each risky asset i has two idiosyncratic cash flow regimes. In the good regime, idiosyncratic cash flows e_t^i grow with low volatility, while in the bad regime they decline with high volatility. There are two macroeconomic regimes as well. In the good regime, aggregate consumption C_t and systematic cash flows $\tilde{D}_t^i$ of asset i grow with low volatility, while in the bad regime they decline with high volatility. Good regime is a natural state of a growing firm or an economy, while a bad regime is a state in which one or more disruptions induce cash flow volatility and decline. $F_{t,s}^i$ is the time-t subjective belief of the investor about the likelihood of asset i being in good idiosyncratic regime at a future point s. $F_{t,s}$ is the time-t subjective belief of the investor about the likelihood of the economy being in the good regime at a future point s. Changes in $F_{t,s}^i$ and $F_{t,s}$ are unpredictable: the beliefs investors expect to have tomorrow about future point s equal their beliefs about s today.

Using math notation, the above can be succinctly stated as follows. Let (idiosyncratic) regime R_1 denote a good regime and let (idiosyncratic) regime R_2 denote a bad regime. Then, for $j = 1, 2$, the dynamics of e_t^i in the idiosyncratic regime R_j are described by

$$\frac{de_t^i}{e_t^i} = \mu_{e,R_j}^i dt + \sigma_{e,R_j}^i dB_{e,R_j,t}^i,$$

and the dynamics of $\tilde{D}_t^i$ and C_t in the macroeconomic regime R_j are described by

$$\frac{d\tilde{D}_t^i}{\tilde{D}_t^i} = \mu_{\tilde{D},R_j}^i dt + \sigma_{\tilde{D},R_j}^i dB_{\tilde{D},R_j,t}^i,$$
$$\frac{dC_t}{C_t} = \mu_{C,R_j} dt + \sigma_{C,R_j} dB_{C,R_j,t},$$

$\mu^i_{e,R_1}, \mu^i_{\tilde{D},R_1}, \mu_{C,R_1} > 0$, $\mu^i_{e,R_2}, \mu^i_{\tilde{D},R_2}, \mu_{C,R_2} < 0$, $\sigma^i_{e,R_1} < \sigma^i_{e,R_2}$, $\sigma^i_{\tilde{D},R_1} < \sigma^i_{\tilde{D},R_2}$, $\sigma_{C,R_1} < \sigma_{C,R_2}$. $B^i_{e,R_j,t}$, $B^i_{\tilde{D},R_j,t}$, $B_{C,R_j,t}$ are corresponding Brownian Motions, where $B^i_{e,R_j,t}$ is uncorrelated with any other Brownian Motion. Subjective regime beliefs are dynamically consistent:

$$E^I_t\left[F^i_{u,s}\right] = F^i_{t,s} \quad t < u < s,$$
$$E^I_t\left[F_{u,s}\right] = F_{t,s} \quad t < u < s,$$

where $E^I_t[.]$ denotes subjective expectation operator. The dynamics of $F^i_{t,s}$ (and of $F_{t,s}$) across t and for fixed s are time-varying and not known to investors in advance. Instead, perceived regime dynamics at time t are implicit in $\left\{F^i_{t,s}\right\}_s$ and $\{F_{t,s}\}_s$, which summarizes all subjective regime-relevant information available at time t about future infinite horizon.

2.1.1 Why Two Types of Regimes Matter

Besides accounting for the obvious fact that regimes can be both macroeconomic and asset-level, incorporating regime-switching in systematic as well as idiosyncratic cash flows has an additional advantage. While bad macroeconomic regimes generally have negative effect on risky assets, some firms can benefit in downturns due to the unique nature of their products and services. For instance, defense contractors can profit in wartime and Pharmaceutical firms can profit during pandemics. In a model with macroeconomic regimes only, accounting for this fact is challenging. With the two types of regimes, however, the increased profitability of positively exposed firms in bad times can be accounted for by a substantial rise in idiosyncratic cash flows that outweigh declines in systematic profitability. The increased profitability in such cases is idiosyncratic in the sense that it is driven by the unique nature of firm products and services rather than the state of the macroeconomy. During covid, for instance, the demand for vaccines was driven by health needs and would have been high irrespective of the state of the economy. Likewise, in wars, defense contractors would experience a substantial rise in cash flows because of the defense needs and irrespective of macroeconomic conditions. The state of the economy would of course still affect the overall profitability of positively exposed firms (e.g. wartime defense spending in a boom would likely be higher than in a recession), but such effect would be reflected in a systematic cash flow stream.

2.2 The Nature of Regime Beliefs

The model treats $\left\{F^i_{t,s}\right\}_s$ and $\{F_{t,s}\}_s$ as model primitives and abstracts away from the belief formation mechanism. While the framework can accommodate specific learning mechanisms, the generality of $\left\{F^i_{t,s}\right\}_s$ and $\{F_{t,s}\}_s$ is a natural starting point since it illustrates the implications of novel model design up to the particulars of a specific learning mechanism.

For VIX-based calibration, one could intuitively interpret regime beliefs in the spirit of the Rational Inattention theory of Sims (2003). It posits that in a world with abundance of information and scarcity of attention, agents cannot process all available or acquirable information and attend to pieces they find most relevant. As a result, objective probabilities are infeasible to know and subjective beliefs reflect the best efforts of the market at forecasting future outcomes. Concluding remarks briefly discuss how this idea can be formally incorporated in the model in a manner consistent with VIX-based calibration.

There are two types of dynamics $F_{t,s}$ exhibits: variation of $F_{t,s}$ over s for fixed t and variation of $F_{t,s}$ over t for fixed s. The first type of variation indicates how time$-t$ subjective market beliefs differ across horizons. If t and s index months, for instance, $F_{t,t+1}$ and $F_{t,t+12}$ would indicate time$-t$ subjective probability of being in the good regime one month from now and one year from now, respectively. The second type of variation indicates how subjective market beliefs about a fixed future point evolve over time. For instance, if t denotes the end of September 2025, $F_{t+1,t+12}$ and $F_{t+2,t+12}$ would denote, respectively, October 2025 and November 2025 subjective beliefs about being in the good regime at the end of September 2026. *The model posits that the stochastic nature of the second kind of variation cannot be known in advance and thus cannot be incorporated into asset prices. This is in contrast to existing models, which specify the stochastic nature of belief evolution over time and assume it is known to investors.*

2.2.1 Assumptions on Regime Beliefs

The model solution further relies on the following two mild assumptions on subjective regime beliefs.

Assumption 1. *There exists a partition* $(t,\infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1},T_p)\}$ *such that* $F_{t,u} = F_t(p)$ *and* $F^i_{t,u} = F^i_t(p)$ *for all* $u \in (T_{p-1},T_p)$, *where* N_{par} *is the number of sets in the partition,* $T_0 = t$ *and* $T_{N_{par}} = \infty$.

Assumption 1 states that infinite future horizon can be decomposed into a finite number of disjoint time segments such that regime beliefs are the same for each point within a segment. Because the decomposition is arbitrary, Assumption 1 introduces no loss of generality from mathematical perspective (and all closed from results in subsequent sections are derived for such arbitrary partition). The choice of the exact partition will in general be informed by data availability or the nature of the problem the model is applied to. For instance, for illustrating a particular mechanism, a 3-segment partition might suffice to account for differences in near-term, medium-term and long-term beliefs.

For calibration, the nature of partition would generally be informed by data availability. For instance, VIX-based calibration methodology naturally pins down the partition. Because VIX measures implied volatility over the next month, there is no way of distinguishing the relative bad regime likelihood the market associates to different disjoint segments within the the month. Hence, assuming t indexes months, the natural choice for T_1 is $t+1$. Because VIX futures are available for each month up to 1-year horizon, one can set $T_k = t+k$ for $k \leq 12$. However, beyond 1 year, VIX futures are not available as densely and the length of (T_{p-1}, T_p) time segments must gradually increase. Such gradual increase in segment length is also in line with intuition: the further we go into the future, harder it becomes to differentiate between the riskiness of adjacent time periods. For instance, the beliefs investors would have about year 6 from now would not likely be meaningfully different from the beliefs they would have about year 7. Hence, it makes sense to assume that regime beliefs are the same for any point in time in between year 6 and 7.

Assumption 2. *Let $M_{u,t}$ denote a random variable that discounts time u cash flows to time t. Subjective covariance from the Investor's perspective between future macroeconomic regime beliefs $F_{u,u}$ and the realized stochastically discounted value of a systematic time-u cash flow $M_{u,t}\tilde{D}^i_u$ of asset i equals zero:*

$$Cov^I_t\left(F_{u,u}, M_{u,t}\tilde{D}^i_u\right) = 0.$$

Likewise, the subjective covariance between future idiosyncratic regime beliefs $F^i_{u,u}$ and the realized stochastically discounted value of an idiosyncratic time-u cash flow $M_{u,t}e^i_u$ of asset i equals zero:

$$Cov^I_t\left(F^i_{u,u}, M_{u,t}e^i_u\right) = 0.$$

To see the intuition behind Assumption 2 cleanly, note that

$$Cov_t^I\left(F_{u,u}, M_{u,t}\tilde{D}_u^i\right) = E_t^I\left[\left(F_{u,u} - F_{t,u}\right)\left(M_{u,t}\tilde{D}_u^i - E_t^I\left[M_{u,t}\tilde{D}_u^i\right]\right)\right].$$

$M_{u,t}\tilde{D}_u^i$ can deviate from $E_t^I\left[M_{u,t}\tilde{D}_u^i\right]$ either because of $\tilde{D}_u^i$ or $M_{u,t}$. For instance, if the deviation is positive, it could be because systematic cash flows $\tilde{D}_u^i$ are high or aggregate consumption growth is low (per Section 2.4, $M_{u,t}$ is inversely related to consumption growth). Because the two possibilities send conflicting signals about systematic bad regime risk, investors do not expect the realized value of $M_{u,t}\tilde{D}_u^i$ alone to be informative about time$-u$ systematic bad regime risk about a small time interval right after u. Similarly,

$$Cov_t^I\left(F_{u,u}^i, M_{u,t}e_u^i\right) = E_t^I\left[\left(F_{u,u}^i - F_{t,u}^i\right)\left(M_{u,t}e_u^i - E_t^I\left[M_{u,t}e_u^i\right]\right)\right]$$

and $M_{u,t}e_u^i$ can be above (below) $E_t^I\left[M_{u,t}e_u^i\right]$ either because e_u^i is high (low) or aggregate consumption growth is low (high). Because e_u^i can be informative only about idiosyncratic bad regime risk, while $M_{u,t}$ - only about the systematic bad regime risk, investors do not expect the realized value of $M_{u,t}e_u^i$ alone to be informative about time$-u$ idiosyncratic bad regime risk about a small time interval right after u. Note that Assumption 2 is directly testable using investor surveys.

2.2.2 New Concept of Regimes

In most existing regime-switching models with Markov-Chain structure, all instances of regimes are ex-ante identical and have the same expected duration. In contrast, the new framework makes no assumptions about the intensity and duration of the regimes assets or economy can experience. Instead, the expected intensity and duration of regimes varies across time and horizon and are determined by subjective regime beliefs. For instance, $F_{t,s}$ that is low throughout the next month ($F_{t,s}$ is low for $s \leq t+1$) but considerably higher afterwards, indicates that the market does not expect a given disruption to be persistent. In contrast, $F_{t,s}$ that is low over the next year indicates that the market is concerned about a persistent disruption. Hence, in the new model, the nature of bad regimes markets might anticipate can be anything from transient and mild to persistent and disastrous. In particular, the market need not necessarily expect by model design that any given disruption will last long or reverse in the long run.

Since $F_{t,s}$ is inferred from VIX data, the same mechanism could also be described

intuitively in terms of VIX: if VIX spikes considerably but VIX futures remain low, one can infer that the market does not expect a current disruption to last long. In contrast, if VIX spikes considerably along with VIX futures, one can infer that the market is expecting a disruption to be persistent.

2.3 Optimal Consumption Allocation

Stochastic Discount Factor is determined by the representative investor maximizing lifetime utility of consumption. The investor solves

$$\max_{\{C_s^I\}_{s=t}^{\infty}} E_t^I \left[\int_t^{\infty} e^{-\delta(s-t)} \frac{\left(C_s^I\right)^{1-\gamma}}{1-\gamma} ds \right]$$

subject to the budget constraint

$$W_t + E_t^I \left[\int_t^{\infty} \frac{M_s}{M_t} Y_s ds \right] \geq E_t^I \left[\int_t^{\infty} \frac{M_s}{M_t} C_s^I ds \right],$$

where δ is time discounting parameter, $E_t^I[.]$ denotes time$-t$ subjective expectation of the representative investor, and M_s is a market stochastic discount factor process. W_t is time-t financial wealth, $E_t^I \left[\int_t^{\infty} \frac{M_s}{M_t} Y_s ds \right]$ is the time$-t$ value of lifetime non-portfolio income and $E_t^I \left[\int_t^{\infty} \frac{M_s}{M_t} C_s^I ds \right]$ is the cost of consumption plan $\{C_s^I\}_{s=t}^{\infty}$. Superscript I indicates that C_s^I is a choice variable of the representative investor as opposed to time$-s$ exogenous aggregate consumption.

2.4 Equilibrium

By the first order conditions of the agent's problem (see the Appendix for details),

$$\frac{M_s}{M_t} = e^{-\delta(s-t)} \left(\frac{C_s^I}{C_t^I} \right)^{-\gamma} \tag{2.1}$$

and by market clearing,

$$C_s^I = C_s \quad \forall s \geq t.$$

The time$-t$ price of a claim to time-s dividend payout D_s^i of asset i is thus given by

$$P_t(D_s^i) = E_t^I \left[e^{-\delta(s-t)} \left(\frac{C_s}{C_t} \right)^{-\gamma} D_s^i \right]$$

and the price of all future cash flows from asset i is given by

$$P_t^i = \int_t^\infty P_t(D_s^i)ds.$$

2.5 New Solution Concept

If investors are not assumed to know the dynamics of regime beliefs and $\left\{F_{t,s}^i\right\}_s$ is arbitrary, how can the model expectations be computed? Decompose the time interval from t to s into finite number of segments of length Δ and assume regime is determined at the beginning of the segment and lasts for sure until its end. Over such segments, the fundamentals are guaranteed to follow simple drift-diffusion process of one of the two regimes, which facilitates closed-form computations of expectations using the recursive application of the Law of Iterated Expectations. Let $P_{t,s}^i(\Delta)$ denote the time$-t$ price of D_s^i under the $\Delta-$decomposition. Then, actual price in the model is given by

$$P_t(D_s^i) = \lim_{\Delta \to 0} P_{t,s}^i(\Delta).$$

In the limit and from the perspective of investors, regimes persist for sure over infinitesimally small intervals only. Hence, there are effectively no restrictions on perceived regime dynamics.

3 Model Solution

3.1 Cash Flow Growth Expectations

3.1.1 Systematic Cash Flows

Consider an expression of the form

$$\mu_{\tilde{D},t}^i(p) \equiv F_t(p)\,\mu_{R_1,\tilde{D}}^i + (1 - F_t(p))\,\mu_{R_2,\tilde{D}}^i,$$

where $\mu_{R_j,\tilde{D}}^i$ is the instantaneous expected growth rate of systematic cash flows of asset i in regime R_j, $j = 1,2$. Because $F_t(p)$ is the subjective probability of being in the good macroeconomic regime (R_1) at any point within (T_{p-1}, T_p), $\mu_{\tilde{D},t}^i(p)$ is simply a probability-weighted average of the instantaneous growth rates corresponding to the two regimes. In standard continuous-time finance with only one regime, the instantaneous

cash flow growth rate $\mu^i_{R_j,\tilde{D}}$ equals the log of the expected one-period gross cash flow growth rate. Similarly, in the framework of this paper, $\mu^i_{\tilde{D},t}(p)$ equals the log of the expected one-period gross cash flow growth rate over (T_{p-1}, T_p). Hence, unlike the standard framework, instantaneous cash flow growth rate $\mu^i_{\tilde{D},t}(p)$ varies across time and horizon due to perceived bad regime risk over different time periods in the future. The proposition that follows formalizes the result.

Proposition 3.1. *Consider a partition* $(t,\infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1}, T_p)\}$ *such that* $F_{t,u} = F_t(p)$ *for all* $u \in (T_{p-1}, T_p)$, *where* N_{par} *is the number of sets in the partition,* $T_0 = t$ *and* $T_{N_{par}} = \infty$. *Let* N_s *denote the number of sets in the partition that intersect* (t,s) *and set* $T_{N_s} = s$. *Then, the subjective expected gross growth rate of systematic cash flows of asset* i *from time* t *to time* s *is described by*

$$E^I_t\left[\frac{\tilde{D}^i_s}{\tilde{D}^i_t}\right] = \prod_{l=1}^{N_s} e^{(T_l - T_{l-1})\mu^i_{\tilde{D},t}(l)},$$

where $\mu^i_{\tilde{D},t}(l)$ *is a log of the expected one-period gross cash flow growth rate over* (T_{l-1}, T_l).

Remark 3.1. The expected gross growth rate expression can alternatively be written as $E^I_t\left[\frac{\tilde{D}^i_s}{\tilde{D}^i_t}\right] = e^{(s-t)\bar{\mu}^i_{\tilde{D},t,s}}$, where $\bar{\mu}_{\tilde{D},t,s} \equiv \sum_{l=1}^{N_s} w_l \mu^i_{\tilde{D},t}(l)$, $w_l = \frac{T_l - T_{l-1}}{s-t}$, $\sum_{l=1}^{N_s} w_l = 1$. In that case, $\bar{\mu}^i_{\tilde{D},t,s}$ can be interpreted as the log of the expected one-period gross cash flow growth rate from time t to time s. It is simply a weighted average of the logs of expected cash flow growth rates over individual time segments $\{(T_{p-1}, T_p)\}_{p=1}^{N_s}$, where the weights correspond to the length of the time segments relative to the length of the whole interval (s,t).

3.1.2 Relation to the Standard Framework

Proposition 3.1 and the results of the following sections extend the standard framework in an intuitive way. With only a single regime R_1, the expected growth rate implied by Proposition 3.1 is described by the standard continuous-time formula

$$E^I_t\left[\frac{\tilde{D}^i_s}{\tilde{D}^i_t}\right] = e^{(s-t)\mu_{\tilde{D},R_1}},$$

which arises because regime beliefs are the same across all future horizons and equal one ($N_{par} = N_s = 1$, $F_t(p) = 1$ for all p, $\mu^i_{\tilde{D},t}(p) = \mu^i_{R_1,\tilde{D}}$). With two regimes but horizon-

invariant beliefs ($F_{t,s} = F_t$ for all $s > t$), the expected growth rate is described by an extension of the standard formula in which the net growth rate $\mu_{\tilde{D},R_1}$ is replaced by the regime-belief-weighted average of the net growth rates in the two regimes denoted by $\mu_{\tilde{D},t}$:

$$E_t^I\left[\frac{\tilde{D}_s^i}{\tilde{D}_t^i}\right] = e^{(s-t)\mu^i_{\tilde{D},t}}. \tag{3.1}$$

The formula arises because beliefs can change anytime but they are uniform for all future points ($N_{par} = N_s = 1,\ F_t(p) = F_t$ for all p, $\mu^i_{\tilde{D},t} = F_t\mu^i_{R_1,\tilde{D}} + (1-F_t)\mu^i_{R_2,\tilde{D}}$). When beliefs are time-varying and differ across two horizons (e.g. investor beliefs over (t,T_1) potentially differ from beliefs over (T_1,s)), the expected gross growth rate formula is an intuitive extension of (3.1) given by

$$E_t^I\left[\frac{\tilde{D}_s^i}{\tilde{D}_t^i}\right] = e^{(T_1-t)\mu^i_{\tilde{D},t}(1)}e^{(s-T_1)\mu^i_{\tilde{D},t}(2)}. \tag{3.2}$$

$\mu^i_{\tilde{D},t}(1)$ is the regime-belief-weighted average of the net growth rates in the two regimes given by regime beliefs over $(T_1 - t)$, while $\mu^i_{\tilde{D},t}(2)$ is the regime-belief-weighted average induced by regime beliefs over $(s - T_1)$. Comparing the expected growth rate expression to (3.1), we see that (3.2) is simply the product of expected growth rate from t to T_1 and the expected growth rate from T_1 to s.

More generally with N_s time segments with potentially different beliefs, expected gross growth rate over (t,s) is the product of the expected growth rates over individual time segments. Hence, Proposition 3.1 extends the standard continuous time formula for the expected growth rate by accounting for the possibility of bad regime risk and the term structure of subjective beliefs about it.

3.1.3 Idiosyncratic Cash Flows

The results of Proposition 3.1 and all discussion points readily carry over to idiosyncratic cash flows with the modification that regime beliefs are now asset-specific rather than systematic. More formally, recall that $F_t^i(p)$ denotes the subjective probability of asset i experiencing a declining regime in idiosyncratic cash flows and define an idiosyncratic analogue of $\mu^i_{\tilde{D},t}(p)$ as

$$\mu^i_{e,t}(p) \equiv F_t^i(p)\,\mu^i_{R_1,e} + \left(1 - F_t^i(p)\right)\mu^i_{R_2,e}.$$

Then, a Corollary of Proposition 3.1 for idiosyncratic cash flows can be formalized as follows:

Corollary 3.1. *The subjective expected gross growth rate of idiosyncratic cash flows of asset i from time t to time s is described by*

$$E_t^I\left[\frac{e_s^i}{e_t^i}\right] = \prod_{l=1}^{N_s} e^{(T_l - T_{l-1})\mu_{e,t}^i(l)}.$$

3.2 Spot and Forward Rates on the Zero-Coupon Risk-Free Asset

In analogy with $\mu_{\tilde{D},t}^i(p)$, consider an expression of the form

$$r_t^f(p) \equiv F_t(p)\, r_{R_1}^f + (1 - F_t(p))\, r_{R_2}^f,$$

where $F_t(p)$ is the subjective probability of being in the good macroeconomic regime (R_1) at any point within (T_{p-1}, T_p), and $r_{R_j}^f$ is the risk free rate in a single-regime-R_j economy described by

$$r_{R_j}^f = \delta + \gamma\mu_{j,C} - \frac{1}{2}\gamma(1+\gamma)\sigma_{j,C}^2,\ j = 1,2,$$

where δ is a time discounting parameter, γ_j is a coefficient of risk aversion in regime R_j, and $\mu_{j,C}$ and $\sigma_{j,C}^2$ are the instantaneous growth rate and volatility of consumption growth in regime R_j. As the next proposition shows, $r_t^f(p)$ is a per-period forward rate on a risk-free asset corresponding to future time segment (T_{p-1}, T_p).

Proposition 3.2. *Consider a partition $(t,\infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1}, T_p)\}$ such that $F_{t,u} = F_t(p)$ for all $u \in (T_{p-1}, T_p)$, where N_{par} is the number of sets in the partition, $T_0 = t$ and $T_{N_{par}} = \infty$. Let N_s denote the number of sets in the partition that intersect (t,s) and set $T_{N_s} = s$. Then, the time-t price $P_{t,s}^f$ of a zero-coupon risk-free asset that matures at time s is given by*

$$P_{t,s}^f = \prod_{l=1}^{N_s} e^{-(T_l - T_{l-1})r_t^f(l)},$$

where $r_t^f(1)$ is a one-period time-t spot rate over (t, T_1) and $r_t^f(l)$ is a time-t one-period forward rate over (T_{l-1}, T_l), $l > 1$.

Remark 3.2. The interpretation of $r_t^f(p)$ as a forward rate for $p>1$ is immediate from the pricing expression itself since $\frac{P_{t,T_p}^f}{P_{t,T_{p-1}}^f} = e^{-(T_p-T_{p-1})r_t^f(p)}$. Likewise, $r_t^f(1)$ is the spot rate because $P_{t,s}^f = e^{-(s-t)r_t^f(1)}$ for $t<s\leq T_1$.

Remark 3.3. The price expression can alternatively be written as $P_{t,s}^f = e^{-(s-t)\bar{r}_{t,s}^f}$ where $\bar{r}_{t,s}^f \equiv \sum_{l=1}^{N_s} w_l r_t^f(l)$, $w_l = \frac{T_l - T_{l-1}}{s-t}$, $\sum_{l=1}^{N_s} w_l = 1$. In that case, $\bar{r}_{t,s}^f$ can be interpreted as a time$-t$ bond yield of horizon $s-t$.

Forward rate $r_t^f(p)$ is also an expected holding period return from T_{p-1} to T_p of the risk free asset maturing after T_p. This is formalized next.

Proposition 3.3. *The gross holding period return $R_{T_{p-1},T_P}^f(s)$ from T_{p-1} to T_p of the risk free asset with maturity at time $s>T_p$ satisfies*

$$\log R_{T_{p-1},T_P}^f(s) = (T_p - T_{p-1}) r_{T_{p-1}}^f(p) + \\ - \sum_{l=p+1}^{N_s} (T_l - T_{l-1}) \Delta_p r^f(l),$$

and

$$E_t^I\left[\log R_{T_{p-1},T_P}^f(s)\right] = r_t(p)$$

where $\Delta_p r^f(l) \equiv r_{T_p}^f(l) - r_{T_{p-1}}^f(l)$.

Proposition 3.3 says that the holding period return of the risk free asset from time T_{p-1} to T_p is determined by time T_{p-1} spot rate over (T_{p-1},T_p) and the revisions in forward rates over (T_p,s) that took place over the holding period. The result follows from two observations. First, because the revisions in forward rates are driven by the changes in perceived bad regime risk and because changes in the perceived bad regime risk over any fixed future time period are unpredictable, the time-t subjective expected changes in forward rates are zero. Secondly, and for the same reason, the time-t subjective expected future spot rate over (T_{p-1},T_p) equals the current forward rate over (T_{p-1},T_p). The two together imply that expected holding period return over (T_{p-1},T_p) equals the current forward rate.

The fact that $E_t^I\left[r_{T_{p-1}}^f(p)\right] = r_t(p)$ along with Remark 3.3 implies a form of an Expectations Hypothesis (EH) of the term structured of interest rates stated below.

Corollary 3.2. *The current yield $\bar{r}^f_{t,s}$ of a risk free asset maturing at time s is related to future spot rates as follows:*

$$\bar{r}^f_{t,s} = \sum_{l=1}^{N_s} w_l E^I_t \left[r^f_{T_{l-1}}(l) \right].$$

The Corollary 3.2 says that the long-term bond-yield is the average of the expected future short-term spot rates. A striking implication of the model is that the EH need not hold in the data even though it holds subjectively. I show in Section 4.5 that the model-implied observed spot rates exhibit behavior consistent with the empirical evidence reported in the literature.

3.3 Dividend Strips

3.3.1 Systematic Cash Flow Strips

In analogy with $r^f_t(p)$, consider an expression of the form

$$r^i_{\tilde{D},t}(p) \equiv F_t(p)\, r^i_{\tilde{D},R_1} + (1 - F_t(p))\, r^i_{\tilde{D},R_2},$$

where

$$r^i_{\tilde{D},R_j} \equiv r^f_{R_j} + \gamma \rho^i_{R_j} \sigma^i_{\tilde{D},R_j} \sigma_{C,R_j}$$

is the discount rate on systematic cash flows of asset i in a single-regime-R_j economy. Define another quantity $\delta^i_{\tilde{D},t}(p)$ as

$$\delta^i_{\tilde{D},t}(p) \equiv r^i_{\tilde{D},t}(p) - \mu^i_{\tilde{D},t}(p).$$

In the standard continuous-time framework with only one regime, $r^i_{\tilde{D},R_1} - \mu^i_{\tilde{D},R_1}$ is the dividend yield defined generally as a one-period rate that discounts current dividend level to the current price of a future-dated dividend strip. In analogy with the single-regime quantities, $r^i_{\tilde{D},t}(p)$ and $\delta^i_{\tilde{D},t}(p)$ are, respectively, a time$-t$ one-period discount rate and a time-t one-period dividend yield over (T_{p-1}, T_p). The proposition that follows formalizes the result.

Proposition 3.4. *Consider a partition $(t,\infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1}, T_p)\}$ such that $F_{t,u} = F_t(p)$ for all $u \in (T_{p-1}, T_p)$, where N_{par} is the number of sets in the partition, $T_0 = t$ and $T_{N_{par}} = \infty$. Let N_s denote the number of sets in the partition that intersect (t,s)*

and set $T_{N_s} = s$. *Then, the time-t price* $P_t\left(\tilde{D}^i_s\right)$ *of time-s systematic cash flow* $\tilde{D}^i_s$ *is given by*

$$P_t\left(\tilde{D}^i_s\right) = \left(\prod_{l=1}^{N_s} e^{-(T_l - T_{l-1})\delta^i_{\tilde{D},t}(l)}\right)\tilde{D}^i_t.$$

Remark 3.4. The expression can alternatively be $P_t\left(\tilde{D}^i_s\right) = e^{-(s-t)\bar{r}^i_{\tilde{D},t,s}}\tilde{D}^i_t e^{(s-t)\bar{\mu}^i_{\tilde{D},t,s}}$, where $\bar{r}_{\tilde{D},t,s} \equiv \sum_{l=1}^{N_s} w_l r^i_{\tilde{D},t}(l)$, $\bar{\mu}_{\tilde{D},t,s} \equiv \sum_{l=1}^{N_s} w_l \mu^i_{\tilde{D},t}(l)$, $w_l = \frac{T_l - T_{l-1}}{s-t}$, $\sum_{l=1}^{N_s} w_l = 1$. Because $\tilde{D}^i_t e^{(s-t)\bar{\mu}^i_{\tilde{D},t,s}}$ is the expected value of $\tilde{D}^i_s$ by Proposition 3.1, $\bar{r}^i_{\tilde{D},t,s}$ can be interpreted as the per-period rate that discount systematic cash flows of asset i from time s to time t.

Remark 3.5. $r^i_{\tilde{D},t}(p)$ can equivalently be written as $r^f_t(p) + \tilde{\gamma}^i_{\tilde{D},t}(p)$,where $\tilde{\gamma}^i_{\tilde{D},t}(p)$ is the risk premium and is described by

$$\tilde{\gamma}^i_{\tilde{D},t}(p) \equiv F_t(p)\,\gamma\rho^i_{R_1}\sigma^i_{\tilde{D},R_1}\sigma_{C,R_1} + (1 - F_t(p))\,\gamma\rho^i_{R_2}\sigma^i_{\tilde{D},R_2}\sigma_{C,R_2}.$$

To see the intuition behind the expression in greater detail, consider first $s \in (t, T_1)$. In this case,

$$\begin{aligned} P_t\left(\tilde{D}^i_s\right) &= e^{-(s-t)\delta^i_{\tilde{D},t}(1)}\tilde{D}^i_t = \\ &= e^{-(s-t)r^i_{\tilde{D},t}(1)}\tilde{D}^i_t e^{(s-t)\mu^i_{\tilde{D},t}(1)} = \\ &= e^{-(s-t)r^i_{\tilde{D},t}(1)}E^I_t\left[\tilde{D}^i_s\right], \end{aligned}$$

where the last equality holds since $\tilde{D}^i_t e^{(s-t)\mu^i_{\tilde{D},t}(1)}$ is the expected time$-s$ dividend level by Proposition 3.1. From here, the interpretation of $r^i_{\tilde{D},t}(1)$ and $\delta^i_{\tilde{D},t}(1)$ as one-period spot discount rate and spot dividend yield over (t, s) is immediate. For $s \in (T_1, T_2)$,

$$\begin{aligned} P_t\left(\tilde{D}^i_s\right) &= e^{-(T_1-t)\delta^i_{\tilde{D},t}(1)}e^{-(s-T_1)\delta^i_{\tilde{D},t}(2)}\tilde{D}^i_t = \\ &= e^{-(T_1-t)r^i_{\tilde{D},t}(1)}e^{-(s-T_1)r^i_{\tilde{D},t}(2)}E^I_t\left[\tilde{D}^i_s\right]. \end{aligned}$$

Hence, $r^i_{\tilde{D},t}(2)$ and $\delta^i_{\tilde{D},t}(2)$ can be interpreted as forward discount rate and forward dividend yield over (T_1, s). The interpretation for $r^i_{\tilde{D},t}(p)$ and $\delta^i_{\tilde{D},t}(p)$ with $p > 2$ is analogous.

The proposition that follows describes a holding period return of a dividend strip and its decomposition into cash flow and discount rate news.

Proposition 3.5. *A net holding return $r_{T_{p-1},T_P}\left(\tilde{D}^i_s\right)$ from time T_{p-1} to time T_p of a systematic cash flow strip $\tilde{D}^i_s$, $s > T_p$, is described by*

$$r_{T_{p-1},T_p}\left(\tilde{D}^i_s\right) \approx \underbrace{(T_p - T_{p-1})\, r^i_{\tilde{D},T_{p-1}}(p)}_{\text{spot discount rate}} + \underbrace{g^i_{\tilde{D},T_{p-1},T_p} - (T_p - T_{p-1})\, \mu^i_{\tilde{D},T_{p-1}}(p)}_{\text{cash flow growth surprise}} -$$
$$\underbrace{- \sum_{l=p+1}^{N_s} (T_l - T_{l-1})\, \Delta_p r^i_{\tilde{D},T_p}}_{\text{discount rate news}} + \underbrace{\sum_{l=p+1}^{N_s} \left(T_{\bar{l}} - T_{l-1}\right) \Delta_p \mu^i_{\tilde{D},T_p}(l)}_{\text{future cash flow news}},$$

and

$$E^I_t\left[r_{T_{p-1},T_p}\left(\tilde{D}^i_s\right)\right] \approx (T_p - T_{p-1})\, r^i_{\tilde{D},t}(p)$$

where $T_{N_s} \equiv s$, $g^i_{\tilde{D},T_{p-1},T_p}$ is the net growth rate of systematic cash flows from time T_{p-1} to T_p, $\Delta_p r^i_{\tilde{D},T_p}(l) = r^i_{\tilde{D},T_p}(l) - r^i_{\tilde{D},T_{p-1}}(l)$ and $\Delta_p \mu^i_{\tilde{D},T_p}(l)$ is defined similarly.

Remark 3.6. In the first expression, exact equality holds if we replace $r_{T_{p-1},T_p}\left(\tilde{D}^i_s\right)$ with $\log R_{T_{p-1},T_p}\left(\tilde{D}^i_s\right)$ on the LHS and $g^i_{\tilde{D},T_{p-1},T_p}$ with $\log\left(\frac{\tilde{D}^i_{T_p}}{\tilde{D}^i_{T_{p-1}}}\right)$ on the RHS, where $R_{T_{p-1},T_p}\left(\tilde{D}^i_s\right)$ denotes the gross one-period holding return on the dividend strip. Hence, " $\approx$ " is based on the first-order log-approximation of gross rates on both sides.

Remark 3.7. The second expression follows from the first after applying further first-order Taylor approximation $E^I_t\left[\frac{\tilde{D}^i_{T_p}}{\tilde{D}^i_{T_{p-1}}}\right] - 1 \approx (T_p - T_{p-1})\, \mu^i_{\tilde{D},t}(p)$, which follows by Proposition 3.1.

Proposition 3.5 says that the subjective expected net holding period return over (T_{p-1}, T_p) of a dividend strip maturing after T_p approximately equals the discount rate corresponding to (T_{p-1}, T_p). The realized return differs from the subjective expectation due to a cash flow growth surprise over the holding period, and the revisions that occurred over the holding period in subjective discount rates and growth expectations corresponding to time periods after T_p due to changes in bad regime risk perceptions about (T_p, s). Because increases in bad regime risk increases discount rates and decreases growth expectations, the revisions in discount rates and growth expectations of systematic cash flows always affect returns in the same direction.

3.3.2 Idiosyncratic Cash Flow Strips

The results of the previous section readily carry over to idiosyncratic cash flows with two modifications. Firstly, because idiosyncratic cash flows are uncorrelated with aggregate consumption, discount rate on idiosyncratic cash flows equals the risk-free rate. Secondly, because macroeconomic regimes do not affect idiosyncratic cash flows by definition, the growth expectations of idiosyncratic cash flows depend on idiosyncratic regime beliefs, not on systematic regime beliefs. The two propositions below describe idiosyncratic cash flow strip prices and returns.

Proposition 3.6. *Consider a partition* $(t,\infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1},T_p)\}$ *such that* $F^i_{t,u} = F^i_t(p)$ *for all* $u \in (T_{p-1},T_p)$, *where* N_{par} *is the number of sets in the partition,* $T_0 = t$ *and* $T_{N_{par}} = \infty$. *Let* N_s *denote the number of sets in the partition that intersect* (t,s) *and set* $T_{N_s} = s$. *Then, the time-t price* $P_t\left(e^i_s\right)$ *of time-s idiosyncratic cash flow* e^i_s *of asset* i *is given by*

$$P_t\left(e^i_s\right) = \left(\prod_{l=1}^{N_s} e^{-(T_l - T_{l-1})\delta^i_{e,t}(l)}\right) e^i_t,$$

where $\delta^i_{e,t}(l) = r^f_t(p) - \mu^i_{e,t}(p)$.

Proposition 3.7. *A net holding return* $r_{T_{p-1},T_P}\left(e^i_s\right)$ *from time* T_{p-1} *to time* T_p *of an idiosyncratic cash flow strip* e^i_s, $s > T_p$, *is described by*

$$r_{T_{p-1},T_p}\left(e^i_s\right) \approx \underbrace{(T_p - T_{p-1})\, r^f_{T_{p-1}}(p)}_{\text{spot discount rate}} + \underbrace{g^i_{e,T_{p-1},T_p} - (T_p - T_{p-1})\, \mu^i_{e,T_{p-1}}(p)}_{\text{cash flow growth surprise}} -$$
$$\underbrace{- \sum_{l=p+1}^{N_s} (T_l - T_{l-1})\, \Delta_p r^f_{T_p}(l)}_{\text{discount rate news}} + \underbrace{\sum_{l=p+1}^{N_s} \left(T_{\bar{l}} - T_{l-1}\right) \Delta_p \mu^i_{e,T_p}(l)}_{\text{future cash flow news}},$$

and

$$E^I_t\left[r_{T_{p-1},T_p}\left(e^i_s\right)\right] \approx (T_p - T_{p-1})\, r^f_t(p)$$

where $T_{N_s} \equiv s$, g^i_{e,T_{p-1},T_p} *is the net growth rate of idiosyncratic cash flows from time* T_{p-1} *to* T_p, $\Delta_p r^f_{T_p}(l) = r^f_{T_p}(l) - r^f_{T_{p-1}}(l)$ *and* $\Delta_p \mu^i_{e,T_p}(l)$ *is defined similarly.*

The interpretation of the return decomposition is analogous to the case of systematic cash flows. The only difference here is that variation in discount rates and growth expectations now have different sources. Discount rates vary due to variation in systematic bad regime risk, while cash flow growth expectations vary due to variation in

idiosyncratic bad regime risk. Because the two sources of risk are uncorrelated, changes in discount rates and changes in growth expectations can affect returns in opposing directions.

3.3.3 Total Cash Flow Strips

Unsurprisingly, the time-t price of a cash flow strip D_s^i of asset i is the sum of the time$-t$ prices of its systematic and idiosyncratic cash flow strips. Because the total cash flow strip D_s^i is a portfolio of systematic cash flow $\tilde{D}_s^i$ and idiosyncratic cash flow e_s^i, the return of the total cash flow strip is the weighted average of the returns of the two strips, where weights correspond to the time$-t$ prices of each cash flow type relative to the time$-t$ price of the total cash flow strip D_s^i.

3.4 Asset Prices

Proposition 3.8. *Consider a partition* $(t,\infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1}, T_p)\}$ *such that* $F_{t,u} = F_t(p)$ *for all* $u \in (T_{p-1}, T_p)$, *where* N_{par} *is the number of sets in the partition,* $T_0 = t$ *and* $T_{N_{par}} = \infty$. *Then, the time-t price* $P^i_{\tilde{D},t}$ *of the systematic cash flows* $\left\{\tilde{D}_s^i\right\}_{s\geq t}$ *of asset* i *is given by*

$$P^i_{\tilde{D},t} = \sum_{p=1}^{N_{par}} w_p^i \frac{\tilde{D}_t^i}{r^i_{\tilde{D},t}(p) - \mu^i_{\tilde{D},t}(p)},$$

where

$$\sum_{p=1}^{N_{par}} w_p^i = 1.$$

Remark 3.8. The closed form expression for the $p-$th weight is $w_p^i = \left(\prod_{l=1}^{p-1} e^{-L_l \delta_t^i(l)}\right)\left(1 - e^{-L_p \delta_t^i(p)}\right)$, where $\prod_{l=1}^{0} e^{-L_l \delta_t^i(l)} \equiv 1$ and $L_l \equiv T_{l+1} - T_l$ is the length of the $(l+1)$-th set in the partition of (t,∞). All else equal, the weight is larger bigger the length of the time interval (T_{p-1}, T_p) and the closer T_{p-1} is to t.

Remark 3.9. The price expression depends on $\{T_p\}_{p=1}^{N_{par}}$ only through $\{L_p\}_{p=1}^{N_{par}}$, i.e. once the length of each time segment (T_{p-1}, T_p) is fixed, specific values of $\{T_p\}_{p=1}^{N_{par}}$ do not affect $P^i_{\tilde{D},t}$. Hence, even though specific values of T_p change as t varies, the variation does not affect the price expression as long as the lengths of future time segments are fixed (i.e. for any t, future time segments are given by a 1-month horizon from t, a horizon in between month 1 and month 3 from t etc).

The price expression has an intuitive interpretation. With only a single regime R_1, $P^i_{\tilde{D},t}$ is described by the standard continuous-time Gordon growth formula

$$P^i_{\tilde{D},t} = \frac{\tilde{D}^i_t}{r^i_{R_1,\tilde{D}} - \mu^i_{R_1,\tilde{D}}},$$

which arises because regime beliefs are the same across all future horizons and equal one ($N_{par} = 1,\ w^i_1 = 1,\ w^i_p = 0\ for\ p > 1,\ F_{t,s} = 1,\ r^i_{\tilde{D},t}(1) = r^i_{R_1,\tilde{D}},\ \mu^i_{\tilde{D},t}(1) = \mu^i_{R_1,\tilde{D}}$). With two regimes but horizon-invariant beliefs ($F_{t,s} = F_t$ for all $s > t$), $P^i_{\tilde{D},t}$ is described by an extension of the continuous-time Gordon growth formula with time-varying discount and growth rates

$$P^i_{\tilde{D},t} = \frac{\tilde{D}^i_t}{r^i_{\tilde{D},t} - \mu^i_{\tilde{D},t}},$$

which arises because beliefs can change anytime but they are assumed to be uniform for all future points ($N_{par} = 1,\ w^i_1 = 1,\ w^i_p = 0\ for\ p > 1,\ F_{t,s} = F_t,\ r^i_{\tilde{D},t} = F_t r^i_{R_1,\tilde{D}} + (1 - F_t) r^i_{R_2,\tilde{D}}$, and $\mu^i_{\tilde{D},t}$ is defined similarly). When beliefs are time-varying and differ across two horizons (e.g. investor beliefs over (t, T_1) potentially differ from beliefs over (T_1, ∞)),

$$P^i_{\tilde{D},t} = \left(1 - e^{-L_1 \delta^i_t(1)}\right) \frac{\tilde{D}^i_t}{r^i_{\tilde{D},t}(1) - \mu^i_{\tilde{D},t}(1)} + e^{-L_1 \delta^i_t(1)} \frac{\tilde{D}^i_t}{r^i_{\tilde{D},t}(2) - \mu^i_{\tilde{D},t}(2)},$$

where $r^i_{\tilde{D},t}(1)$ is the discount rate for time period (t, T_1), $r^i_{\tilde{D},t}(2)$ is the discount rate for time period (T_1, ∞) and the two potentially differ because of different perceptions of bad regime risk over the two time periods. Likewise, $\mu^i_{\tilde{D},t}(1)$ is the subjective expected growth rate of systematic cash flows over (t, T_1), $\mu^i_{\tilde{D},t}(2)$ is the subjective expected growth rate of systematic cash flows over (T_1, ∞) and the two potentially differ because of different perceptions of bad regime risk over the two time periods. If perceived bad regime risk is higher over (t, T_1), for instance, the discount rate corresponding to (t, T_1) is higher and expected cash flow growth rate is lower. As a result, $r^i_{\tilde{D},t}(1) - \mu^i_{\tilde{D},t}(1)$ would be higher than $r^i_{\tilde{D},t}(2) - \mu^i_{\tilde{D},t}(2)$. Intuitively, one can think of $r^i_{\tilde{D},t}(p) - \mu^i_{\tilde{D},t}(p)$ as a counterfactual dividend-price ratio that would be observed if regime beliefs over any future point were the same as the beliefs over $p-$th time segment, $p = 1, 2$. Similarly, one can think of $\frac{\tilde{D}^i_t}{r^i_{\tilde{D},t}(p) - \mu^i_{\tilde{D},t}(p)}$ as a counterfactual price that would be observed if regime beliefs over any future point were the same as the beliefs over $p-$th time segment.

The actual price is then a weighted average of the two counterfactual prices, where the weight for the first counterfactual price is higher the larger the length L_1 of the first time segment.

These intuitions carry over to the general case with N_{par} time segments with an additional feature: the weights different future time segments receive also depend on the proximity to t: for instance, (t, T_1) would receive higher weight than $(T_1, T_1 + (T_1 - t))$ even though both have the same length. In general, it is useful to define additional variable $\delta^i_{\tilde{D},t}$ as

$$\frac{1}{\delta^i_{\tilde{D},t}} \equiv \sum_{p=1}^{N_{par}} w^i_p \frac{1}{r^i_{\tilde{D},t}(p) - \mu^i_{\tilde{D},t}(p)}.$$

In this case, the price of systematic cash flows can be expressed as

$$P^i_{\tilde{D},t} = \frac{\tilde{D}^i_t}{\delta^i_{\tilde{D},t}}$$

and $\delta^i_{\tilde{D},t}$ can be interpreted as instantaneous dividend-price ratio of systematic cash flows. The only source of time-variation variation in $\delta^i_{\tilde{D},t}$ is regime beliefs and it increases any time subjective bad regime likelihood about any future point in time goes up. An interesting property of the model is that variation in regime beliefs (and hence $\delta^i_{\tilde{D},t}$) is not mechanically tied to cash flow or other economic dynamics and is backed out from implied volatility data instead. Hence, $\delta^i_{\tilde{D},t}$ can capture many pieces of information that cannot be reduced to primitive variables and are thus often neglected in existing asset pricing models. In particular, $\delta^i_{\tilde{D},t}$ can induce price to go in a direction opposite to cash flow growth. Striking implications of this will be discussed in subsequent sections in detail.

A result similar to Proposition 3.8 holds for idiosyncratic cash flows as well. It is considered next.

Proposition 3.9. *Consider a partition* $(t, \infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1}, T_p)\}$ *such that* $F^i_{t,u} = F^i_t(p)$ *for all* $u \in (T_{p-1}, T_p)$, *where* N_{par} *is the number of sets in the partition,* $T_0 = t$ *and* $T_{N_{par}} = \infty$. *Then, the time-t price* $P^i_{e,t}$ *of idiosyncratic cash flows* $\left\{e^i_s\right\}_{s \geq t}$ *of asset* i *is given by*

$$P^i_{e,t} = \sum_{p=1}^{N_{par}} w^i_{e,p} \frac{e^i_t}{r^f_t(p) - \mu^i_{e,t}(p)},$$

where

$$\sum_{p=1}^{N_{par}} w_{e,p}^{i} = 1.$$

Remark 3.10. $w_{e,p}^{i} = \left(\prod_{l=1}^{p-1} e^{-L_l \delta_{e,t}^{i}(l)}\right)\left(1 - e^{-L_p \delta_{e,t}^{i}(p)}\right)$. The expression is the same as the one in Proposition 3.8 up to $\delta_{e,t}^{i}(p)$ term, which in this case equals $\delta_{e,t}^{i}(p) = r_t^f(p) - \mu_{e,t}^{i}(p)$.

The price expression for idiosyncratic cash flows is very similar to the price expression of systematic cash flows and all discussion points readily carry over with two caveats. First, because idiosyncratic cash flows carry no systematic risk, its discount rate for any horizon equals the corresponding risk-free rate. Hence, the discount rate on idiosyncratic cash flows is pro-cyclical. Secondly, while the subjective expected growth rates of systematic cash flows are affected by systematic bad regime risk, the subjective expected growth rates of idiosyncratic cash flows are affected by idiosyncratic bad regime risk. Because the risk-free rate is affected by systematic risk, the price of idiosyncratic cash flows depends both on systematic and idiosyncratic regime risk.

In analogy with $\delta_{\tilde{D},t}^{i}$, it is useful to define $\delta_{e,t}^{i}$. All insights about $\delta_{\tilde{D},t}^{i}$ readily carry over to $\delta_{e,t}^{i}$ except that the latter depends on idiosyncratic regime beliefs instead of systematic ones. I formalize the definitions of the two variables below.

Definition 3.1. Define $\delta_{\tilde{D},t}^{i}$ and $\delta_{e,t}^{i}$ as

$$\frac{1}{\delta_{\tilde{D},t}^{i}} \equiv \sum_{p=1}^{N_{par}} w_p^{i} \frac{1}{r_{\tilde{D},t}^{i}(p) - \mu_{\tilde{D},t}^{i}(p)}$$

and

$$\frac{1}{\delta_{e,t}^{i}} \equiv \sum_{p=1}^{N_{par}} w_{e,p}^{i} \frac{1}{r_t^f(p) - \mu_{e,t}^{i}(p)}.$$

Then, time$-t$ prices of systematic and idiosyncratic cash flows can respectively be expressed as

$$P_{\tilde{D},t}^{i} = \frac{\tilde{D}_t^{i}}{\delta_{\tilde{D},t}^{i}}$$

and

$$P_{e,t}^{i} = \frac{e_t^{i}}{\delta_{e,t}^{i}}.$$

$\delta_{\tilde{D},t}^{i}$ rises any time there is an increase in subjective systematic bad regime likelihood

over any future point in time and vice versa. $\delta^i_{e,t}$ rises any time there is an increase in subjective idiosyncratic bad regime likelihood of asset i over any future point in time and vice versa.

Unsurprisingly, the price of asset i is the sum of the prices of its systematic and idiosyncratic cash flow streams. This is stated formally next and proved in the Appendix.

Proposition 3.10. *Consider a partition* $(t,\infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1}, T_p)\}$ *such that* $F^i_{t,u} = F^i_t(p)$ *and* $F_{t,u} = F_t(p)$ *for all* $u \in (T_{p-1}, T_p)$, *where* N_{par} *is the number of sets in the partition,* $T_0 = t$ *and* $T_{N_{par}} = \infty$. *Then, the time-t price* P^i_t *of asset i is described by*

$$P^i_t = P^i_{\tilde{D},t} + P^i_{e,t},$$

where $P^i_{\tilde{D},t}$ *is the price of systematic cash flows of asset i from Proposition 3.8 and* $P^i_{e,t}$ *is the price of of idiosyncratic cash flows of asset i from Proposition 3.9.*

Remark 3.11. Per Definition 3.1, the time-t price of asset i can be expressed as $P^i_t = \frac{\tilde{D}^i_t}{\delta^i_{\tilde{D},t}} + \frac{e^i_t}{\delta^i_{e,t}}$.

3.5 Model-Implied Future Price Forecasts and Euler Equation

In existing continuous-time asset pricing models, price P_t of a risky asset typically satisfies an Euler Equation equation of the form

$$P_t = E_t\left[M_{t,t+1}P_{t+1} + \int_t^{t+1} M_{t,s}D_s ds\right],$$

where $M_{t,s}$, $s > t$, is an SDF that discounts cash flows from time s to time t. In the framework of this paper, such an equation is not well-defined, since P_{t+1} depends nonlinearly on future regime beliefs, the probability distribution of which is not known to the market. Instead, the framework yields a modified version of Euler Equation in which P_{t+1} is replaced by a market forecast $P^I_{t,t+1}$ of a future price as a function of future cash flow D_{t+1}. Intuitively, $P^I_{t,t+1}$ is what the market expects time $t+1$ price to be for a given future cash flow level D_{t+1} and the current outlook it has about future horizon $(t+1,\infty)$.

To define $P^I_{t,t+1}$ more formally, let $P^I_{t,t+1}(D_s)$, $s > t+1$, denote the subjective expected value of stochastically discounted D_s to time $t+1$ using time$-t$ beliefs about time period $(t+1, s)$:

$$P^I_{t,t+1}(D_s) \equiv E^I_{t,t+1}[M_{s,t+1}D_s].$$

$P^I_{t,t+1}(D_s)$ is what market expects time $t+1$ price of D_s to be for a given level of future cash flow D_{t+1} and given time-t beliefs about $(t+1,s)$. I call $P^I_{t,t+1}(D_s)$ a time-t partial forecast of time $t+1$ price of a claim to D_s. It is a forecast in the sense that $P^I_{t,t+1}(D_s)$ is obtained after replacing time $t+1$ beliefs about $(t+1,s)$ using time$-t$ beliefs about $(t+1,s)$, which are subjectively optimal forecast of time $t+1$ beliefs. $P^I_{t,t+1}(D_s)$ is partial in the sense that it does not forecast time $t+1$ cash flows and gives future price estimate for a given realization of D_{t+1}. Hence, from the perspective of time t, $P^I_{t,t+1}(D_s)$ is a random variable that depends on D_{t+1}. $P^I_{t,t+1}$ is then defined as the sum of partial forecasts of prices of all future cash flows from time $t+1$ onward. The proposition that follows describes a modified version of the Euler Equation that is based on partial forecasts of cash flow strip prices.

Proposition 3.11. *Let P^i_t denote the time-t price of a risky asset i and define time$-t$ subjective partial forecast $P^I_{t,t+1}(i)$ of the time $t+1$ price of asset i as*

$$P^I_{t,t+1}(i) \equiv \int_{t+1}^{\infty} P^I_{t,t+1}\left(D^i_s\right) ds.$$

Then, the following are true:

$$P_t\left(D^i_s\right) = E^I_t\left[M_{t,t+1}P^I_{t,t+1}\left(D^i_s\right)\right],\ s > t+1$$

and

$$P^i_t = E^I_t\left[M_{t,t+1}P^I_{t,t+1}(i) + \int_t^{t+1} M_{t,s}D^i_s ds\right].$$

Proof. See the Appendix. □

3.6 Asset Returns

In general, any cash-flow yielding asset can be thought of as a portfolio of two hypothetical assets - one that is made up of infinite idiosyncratic cash flow stream and another that is made up of infinite systematic cash flow stream. Hence, the return on any asset must equal a weighted average of the returns of its two constituent hypothetical assets, where weights correspond to the ratio of the hypothetical asset value to the total value of the actual asset. Each hypothetical asset is in turn a portfolio of cash flow strips. Hence, the return on the hypothetical asset must equal the weighted average of the returns on individual strips, where the weights correspond to the ratios of strip prices

to the total stream price. This is the insight that connects total asset returns to the individual cash flow strip returns derived in the previous sections.

Throughout this section, I derive expressions for total returns that include dividend payouts. Because dividends are paid out continuously in continuous-time models, total return is not uniquely defined. For instance, one could define one-period return on systematic cash flow stream of asset i as

$$R^i_{\tilde{D},t,t+1} \equiv \frac{P^i_{\tilde{D},t+1} + \int_t^{t+1} \tilde{D}_s ds}{P^i_{\tilde{D},t}},$$

where $\int_t^{t+1} \tilde{D}_s ds$ is a simple sum of all dividends continuously paid out in between t and $t+1$. Alternatively, one could define the return by replacing $\tilde{D}_s$ using its forward value $\tilde{D}_s e^{(t+1-s) r_{t,t+1}\left(\tilde{D}^i_{t+1}\right)}$, which simply converts time-s payouts to their time $t+1$ equivalents at the realized return rate. The derivations that follow rely on the second type of definition.

3.6.1 Return on Systematic Cash Flow Stream

Proposition 3.12. *Consider a partition* $(t,\infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1}, T_p)\}$ *such that* $F_{t,u} = F_t(p)$ *for all* $u \in (T_{p-1}, T_p)$*, where* N_{par} *is the number of sets in the partition,* $T_0 = t$ *and* $T_{N_{par}} = \infty$*. Without loss of generality, set* $T_p - T_{p-1} = 1$ *for all* $p < N_{par}$*. Then, the total net return* $r^i_{\tilde{D},t,T_1}$ *on systematic cash flow stream of asset* i *from time* t *to time* T_1 *is described by*

$$r^i_{\tilde{D},t,T_1} \approx \underbrace{(T_1 - t)\, r^i_{\tilde{D},t}(1)}_{\text{spot discount rate}} + \underbrace{g^i_{\tilde{D},t,T_1} - (T_1 - t)\, \mu^i_{\tilde{D},t,T_1}}_{\text{cash flow growth surprise}} - \underbrace{\sum_{p=2}^{N_{par}} \rho^i_{\tilde{D},t}(p)\, \Delta r^i_{\tilde{D},T_1}(p)}_{\text{discount rate news}} + \underbrace{\sum_{p=2}^{N_{par}} \rho^i_{\tilde{D},t}(p)\, \Delta \mu^i_{\tilde{D},T_1}(p)}_{\text{future cash flow news}}$$

and

$$E^I_t\left[r^i_{\tilde{D},t,T_1}\right] \approx (T_1 - t)\, r^i_{\tilde{D},t}(1),$$

where

$$\rho^i_{\tilde{D},t}(p) = \int_{T_{p-1}}^{T_p} \frac{P_t\left(\tilde{D}^i_s\right)}{P^i_{\tilde{D},t}} (s - T_{p-1})\, ds + \mathbf{1}_{\{p<N_{par}\}} \times \int_{T_2}^{\infty} \frac{P_t\left(\tilde{D}^i_s\right)}{P^i_{\tilde{D},t}} ds,$$

$\Delta r^i_{\tilde{D},T_1}(p) \equiv r^i_{\tilde{D},T_1}(p) - r^i_{\tilde{D},t}(p)$, $\Delta\mu^i_{\tilde{D},T_1}(p)$ *is defined similarly and* $'\approx'$ *relies on the first-order log-approximations.*

Remark 3.12. The assumption $T_p - T_{p-1} = 1$ is made for convenience and is non-restrictive, since $F_{t,u}$ can have the same value over more than one time segment in the partition of (t,∞).

The total return expression is very similar to the expression for holding-period return on systematic cash flow strip maturing after T_1 (see Proposition 3.5). Like with the individual cash flow strips, i) the subjective expected total return of the systematic cash flow stream of asset i from time t to time T_1 approximately equals spot discount rate over (t,T_1) and ii) the realized return deviates from the subjective expectation because of the cash flow growth surprise over (t,T_1) and changes in discount rates and growth expectations corresponding to time periods after T_1. The only difference between the return expressions of the cash flow stream and individual strips arises through the discount rate news and future cash flow growth news terms. Individual cash flow strip maturing at time s is sensitive to revisions in forward rates corresponding to time periods before s only. In contrast, cash flow stream is sensitive to all future forward rates, but the sensitivity is strictly decreasing in horizon ($\rho^i_{\tilde{D},t}(p)$ is strictly decreasing in p). Because asset and cash flow strip prices have fully calibrable closed-form expressions, so does $\rho^i_{\tilde{D},t}(p)$.

3.6.2 Return on Idiosyncratic Cash Flow Stream

The total return on idiosyncratic cash flow stream has analogous expression with the modification that its discount rates equal the risk free rates and systematic regime beliefs are replaced by idiosyncratic regime beliefs. The result is stated formally next.

Proposition 3.13. *Consider a partition* $(t,\infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1},T_p)\}$ *such that* $F^i_{t,u} = F^i_t(p)$ *for all* $u \in (T_{p-1},T_p)$, *where* N_{par} *is the number of sets in the partition,* $T_0 = t$ *and* $T_{N_{par}} = \infty$. *Without loss of generality, set* $T_p - T_{p-1} = 1$ *for all* $1 < p < N_{par}$. *Then, the total net return* r^i_{e,t,T_1} *on idiosyncratic cash flow stream of asset* i *from time* t *to*

time T_1 is described by

$$r^i_{e,t,T_1} \approx \underbrace{(T_1-t)\, r^f_t(1)}_{\text{spot discount rate}} + \underbrace{g^i_{e,t,T_1} - (T_1-t)\,\mu^i_{e,t,T_1}}_{\text{cash flow growth surprise}} - \underbrace{-\sum_{p=2}^{N_{par}} \rho^i_{e,t}(p)\,\Delta r^f_{T_1}(p)}_{\text{discount rate news}} + \underbrace{\sum_{p=2}^{N_{par}} \rho^i_{e,t}(p)\,\Delta\mu^i_{e,T_1}(p)}_{\text{future cash flow news}}$$

and

$$E^I_t\left[r^i_{e,t,T_1}\right] \approx (T_1-t)\, r^f_t(1),$$

where

$$\rho^i_{e,t}(p) = \int_{T_{p-1}}^{T_p} \frac{P_t\left(e^i_s\right)}{P^i_{e,t}}(s-T_{p-1})\,ds + \mathbf{1}_{\{p<N_{par}\}} \times \int_{T_2}^{\infty} \frac{P_t\left(e^i_s\right)}{P^i_{e,t}} ds,$$

$\Delta r^f_{T_1}(p) \equiv r^f_{T_1}(p) - r^f_t(p)$, $\Delta\mu^i_{e,T_1}(p)$ is defined similarly and $'\approx'$ relies on the first-order log-approximations.

Remark 3.13. The assumption $T_p - T_{p-1} = 1$ is made for convenience and is non-restrictive, since $F^i_{t,u}$ can have the same value over more than one time segment in the partition of (t,∞).

3.6.3 Total Asset Returns

Since the total return on asset i equals the value-weighted average of the returns of its systematic and idiosyncratic cash flow streams, the results of the previous sections can be put together to derive the expressions for total asset returns. Let $w^i_t\left(\tilde{D}\right)$ and $w^i_t(e)$ denote, respectively, the ratios of time$-t$ valuations of systematic and idiosyncratic cash flow streams relative to the total value of asset i. Define $r^i_t(p)$, $\bar{g}^i_{t,T_1}$ and $\bar{\mu}^i_{t,T_1}$ as follows:

$$\begin{aligned}
r^i_t(p) &\equiv w^i_t\left(\tilde{D}\right) r^i_{\tilde{D},t}(p) + w^i_t(e)\, r^f_t(p),\\
\bar{g}^i_{t,T_1} &\equiv w^i_t\left(\tilde{D}\right) g^i_{\tilde{D},t,T_1} + w^i_t(e)\, g^i_{e,t,T_1},\\
\bar{\mu}^i_{t,T_1} &\equiv w^i_t\left(\tilde{D}\right) \mu^i_{\tilde{D},t,T_1} + w^i_t(e)\, \mu^i_{e,t,T_1},
\end{aligned}$$

$w^i_t(e) = \frac{P^i_{e,t}}{P^i_t}$, $w^i_t\left(\tilde{D}\right) = \frac{P^i_{\tilde{D},t}}{P^i_t}$. $r^i_t(p)$ is the value weighted average of the discount rates on systematic and idiosyncratic cash flows, and $\bar{g}^i_{t,T_1}$ and $\bar{\mu}^i_{t,T_1}$ have analogous inter-

pretations. Recall from Remark 3.5 that the time-t forward discount rate $r^i_{\tilde{D},t}(p)$ of the systematic cash flows of asset i corresponding to (T_{p-1}, T_p) can be written as $r^f_t(p) + \tilde{\gamma}^i_{t,\tilde{D}}(p)$,where $r^f_t(p)$ is the forward risk-free rate and $\tilde{\gamma}^i_{t,\tilde{D}}(p)$ is the subjective risk premium on systematic cash flows corresponding with time period (T_{p-1}, T_p). $r^i_t(p)$ can thus also be expressed as

$$\begin{aligned} r^i_t(p) &= r^f_t(p) + w^i_t\left(\tilde{D}\right)\tilde{\gamma}^i_{t,\tilde{D}}(p) \equiv \\ &\equiv r^f_t(p) + \tilde{\gamma}^i_t(p), \end{aligned}$$

where $\tilde{\gamma}^i_t(p)$ is the risk premium on asset i. The proposition that follows describes the total return of asset i from time t to time T_1.

Proposition 3.14. *Consider a partition* $(t,\infty) \equiv \bigcup_{p=1}^{N_{par}} \{(T_{p-1}, T_p)\}$ *such that* $F_{t,u} = F_t(p)$ *and* $F^i_{t,u} = F^i_t(p)$ *for all* $u \in (T_{p-1}, T_p)$, *where* N_{par} *is the number of sets in the partition,* $T_0 = t$ *and* $T_{N_{par}} = \infty$. *Without loss of generality, set* $T_p - T_{p-1} = 1$ *for all* $1 < p < N_{par}$. *Then, the total net return* r^i_{t,T_1} *of asset i from time t to time* T_1 *is described by*

$$\begin{aligned} r^i_{t,T_1} \approx\ & \underbrace{(T_1 - t)\, r^i_t(1)}_{\text{spot discount rate}} + \underbrace{\bar{g}^i_{t,T_1} - (T_1 - t)\,\bar{\mu}^i_{t,T_1}}_{\text{cash flow growth surprise}} - \\ & \underbrace{-\sum_{p=2}^{N_{par}} \rho^f_t(p)\,\Delta r^f_{T_1}(p)}_{\text{risk free rate news}} \underbrace{-\sum_{p=2}^{N_{par}} \bar{\rho}^i_{t,\tilde{D}}(p)\,\Delta\tilde{\gamma}^i_{\tilde{D},T_1}(p) +}_{\text{risk premium news}} \\ & \underbrace{+\sum_{p=2}^{N_{par}} \bar{\rho}^i_{t,\tilde{D}}(p)\,\Delta\mu^i_{\tilde{D},T_1}(p)}_{\text{systematic cash flow news}} + \underbrace{\sum_{p=2}^{N_{par}} \bar{\rho}^i_{t,e}(p)\,\Delta\mu^i_{e,T_1}(p)}_{\text{idiosyncratic cash flow news}} \end{aligned}$$

and

$$E^I_t\left[r^i_{t,T_1}\right] \approx (T_1 - t)\, r^i_t(1)$$

where the weights $\rho_t^f(p)$, $\bar{\rho}_{t,\tilde{D}}^i(p)$ *and* $\bar{\rho}_{t,e}^i(p)$ *are defined as follows*

$$\rho_t^f(p) = \int_{T_{p-1}}^{T_p} \frac{P_t\left(D_s^i\right)}{P_t^i}(s-T_{p-1})\,ds + \mathbf{1}_{\{p<N_{par}\}} \times \int_{T_2}^{\infty} \frac{P_t\left(D_s^i\right)}{P_t^i} ds,$$
$$\bar{\rho}_{t,\tilde{D}}^i(p) = \int_{T_{p-1}}^{T_p} \frac{P_t\left(\tilde{D}_s^i\right)}{P_t^i}(s-T_{p-1})\,ds + \mathbf{1}_{\{p<N_{par}\}} \times \int_{T_2}^{\infty} \frac{P_t\left(\tilde{D}_s^i\right)}{P_t^i} ds,$$
$$\bar{\rho}_{t,e}^i(p) = \int_{T_{p-1}}^{T_p} \frac{P_t\left(e_s^i\right)}{P_t^i}(s-T_{p-1})\,ds + \mathbf{1}_{\{p<N_{par}\}} \times \int_{T_2}^{\infty} \frac{P_t\left(e_s^i\right)}{P_t^i} ds,$$

$\Delta r_{T_1}^f(p) \equiv r_{T_1}^f(p) - r_t^f(p) = \Delta F_{T_1}(p)\left(r_{R_1}^f - r_{R_2}^f\right)$, *other 'delta terms' are defined similarly and* $'\approx'$ *relies on the first-order log-approximations of gross rates of change.*

Remark 3.14. Note the subtle difference between $\bar{\rho}_{t,\tilde{D}}^i(p)$ above and $\rho_{t,\tilde{D}}^i(p)$ in Proposition 3.12. The former is scaled by P_t^i, while the latter - $P_{\tilde{D},t}^i$. This is because the weights in Proposition 3.12 are relative to the value of the systematic cash flow stream, whereas the weights in this proposition are relative to the total asset value. The difference between $\bar{\rho}_{t,e}^i(p)$ and $\rho_{t,e}^i(p)$ is analogous.

Remark 3.15. $\rho_t^f(p) = \bar{\rho}_{t,\tilde{D}}^i(p) + \bar{\rho}_{t,e}^i(p)$. All three terms are strictly decreasing in p because cash flow strip prices strictly decrease with increasing horizon.

The proposition above says that the subjective expected return on asset i equals the discount rate over (t,T_1), which is in turn the sum of the risk-free rate and the risk premium on total cash flows from asset i over (t,T_1). The realized return deviates from the subjective expectation because of cash flow growth surprise over (t,T_1), and the revisions that took place over (t,T_1) in discount rates and growth expectations of systematic and idiosyncratic cash flows corresponding to time periods after T_1. The revisions in idiosyncratic cash flow expectations are driven by changes in idiosyncratic bad regime risk, while revisions in systematic cash flow expectations are driven by changes in systematic bad regime risk. When post-T_1 systematic bad regime risk increases in between t and T_1, risk free rates go down and risk premiums go up. The net effect of the discount rate news depends on the exposure of asset i to macroeconomic risk described by $\bar{\rho}_{t,\tilde{D}}^i(p)$ terms - higher they are, the stronger the effect of the risk premium. Because discount rates and growth expectations change only due to changes in perceived bad regime risk and because changes in perceived bad regime risk about fixed future time periods are unpredictable to the investor, the revisions in the discount and growth rates have zero mean subjectively. A striking implication of Proposition

3.14 is that observed average returns need not equal the average discount rate in the data even though the subjective expected return always equals the prevailing discount rate (see details in Section 4.3).

3.6.4 Campbell-Shiller Decomposition

Proposition 3.14 is a model-specific expression for the decomposition of returns into cash flow and discount rate news, which is a log-approximation to an accounting identity and does not aim to explain the sources of the two news types. In contrast, Proposition 3.14 i) provides a theory of variation of discount rates and growth expectations and ii) further decomposes discount rate news into the risk-free rate news and risk premium news, and cash flow news - into systematic cash flow news and idiosyncratic cash flow news.

3.7 Calibration Methodology

4 Implications

4.1 Price Crashes in Good Times and Fast Price Recoveries in Bad Times

Recall from Remark 3.11 that the time$-t$ price P_t^i of asset i can be compactly expressed as

$$P_t^i = \frac{\tilde{D}_t^i}{\delta^i_{\tilde{D},t}} + \frac{e_t^i}{\delta^i_{e,t}},$$

where $\tilde{D}_t^i$ $\left(e_t^i\right)$ is a current level of systematic (idiosyncratic) cash flows, and $\delta^i_{\tilde{D},t}$ $\left(\delta^i_{e,t}\right)$ is a function of systematic (idiosyncratic) regime beliefs and increases any time there is an increase in systematic (idiosyncratic) bad regime likelihood about any future point of time. Because $\delta^i_{\tilde{D},t}$ and $\delta^i_{e,t}$ are not mechanically tied to cash flow levels, prices can crash in anticipation of bad times even if they do not materialize and can start recovering even if cash flows are still in decline. The latter phenomenon was particularly evident during Covid, when the market sharply rebounded in Spring 2020 even though the economy was still in decline.

The model facilitates the above mechanisms because it allows regime beliefs to move independently from cash flow realizations. Hence, the two variables can affect prices in

different directions. Such phenomena is harder to observe in existing models in which investor beliefs coincide with true dynamics (even up to a few parameters). Because investors know (or think to know) the true consumption dynamics, their beliefs about consumption growth cannot worsen (improve) until they observe declines (increases) in consumption growth.

In fact, Ghosh et al. (2017) estimate the SDF non-parametrically and conclude that "winner" consumption-based asset pricing models (e.g. habit persistence and long-run risk models) and "most of the more recent models" would benefit from augmentation with a component that generates market crashes without macroeconomic contractions. This is exactly what the paper accomplishes with a carefully motivated microfoundation and applies to macro as well as firm-level events (the former through $\delta^i_{\tilde{D},t}$ and the latter through $\delta^i_{e,t}$).

4.2 Yield Curve Inversion/Reversions Without a Regime Shift

Recall from Section 3.2 that the forward one-period risk free rate corresponding to time period (T_{p-1}, T_p) is described by

$$r^f_t(p) \equiv F_t(p)\, r^f_{R_1} + (1 - F_t(p))\, r^f_{R_2},$$

where $r^f_{R_j}$ is the risk-free rate in a single-regime-R_j world. Because $r^f_{R_1} > r^f_{R_2}$, increase in the bad regime likelihood over (T_{p-1}, T_p) decreases $r^f_t(p)$ irrespective of the current state of the economy. In terms of the term structure, $r^f_t(p)$ is increasing in p if and only if $F_t(p)$ is increasing in p, i.e. the risk free rate yield curve is upward-sloping as long as time$-t$ future outlook is incrementally improving in horizon. Conversely, the yield curve is downward-sloping over a given segment as long as future outlook is incrementally worsening in horizon. Because future outlook $F_t(p)$ over (T_{p-1}, T_p) is not mechanically tied to the state of the economy in the model, $\{F_t(p)\}_p$ can change in response to abstract news any time. Hence, news arrival in good times that induces the market to expect a recession in the near term will invert the yield curve. Likewise, the news arrival in an economic downturn that induces the market to expect a recovery in the near term will revert the yield curve.

4.3 Growth Component of Average Returns

Mehra and Prescott (1985) show that the market risk premium in a simple Lucas Tree model is much lower than the observed average excess returns. Since then, the finance literature has tried to explain away the puzzle by modeling additional sources of risk (e.g. long-run risk, disaster risk, habit persistence etc) that make investor risk premium higher. While the framework of this paper has the same flavor (because the possibility of bad regimes results in a higher risk premium), it also offers a new insight: the risk premium need not equal the average realized excess returns in the data even though it always equals the subjective expected excess return of the investor. In particular, I show below that growing firms will exhibit stock returns above the investor risk premium because of gradually improving future outlook and the compensation for unrealized risk.

To see this, recall from Proposition 3.14 that the excess return of asset i from time t to time T_1 is described by

$$\begin{aligned} r^i_{t,T_1} - \underbrace{(T_1-t)\, r^f_t(1)}_{spot\ risk\ free\ rate} \approx \underbrace{(T_1-t)\, \tilde{\gamma}^i_t(1)}_{spot\ risk\ premium} + \underbrace{\bar{g}^i_{t,T_1} - (T_1-t)\, \bar{\mu}^i_{t,T_1}}_{cash\ flow\ growth\ surprise} - \\ \underbrace{- \sum_{p=2}^{N_{par}} \rho^f_t(p)\, \Delta r^f_{T_1}(p)}_{risk\ free\ rate\ news} \underbrace{- \sum_{p=2}^{N_{par}} \bar{\rho}^i_{t,\tilde{D}}(p)\, \Delta \tilde{\gamma}^i_{\tilde{D},T_1}(p) +}_{risk\ premium\ news} \\ \underbrace{+ \sum_{p=2}^{N_{par}} \bar{\rho}^i_{t,\tilde{D}}(p)\, \Delta \mu^i_{\tilde{D},T_1}(p) +}_{systematic\ cash\ flow\ news} \underbrace{\sum_{p=2}^{N_{par}} \bar{\rho}^i_{t,e}(p)\, \Delta \mu^i_{e,T_1}(p)}_{idiosyncratic\ cash\ flow\ news}, \end{aligned}$$

where the cash flow growth surprise has zero subjective mean by construction, and so do all "delta terms" because they are linear functions of subjectively expected belief changes about fixed future time periods. For instance,

$$\begin{aligned} E^I_t\left[\Delta r^f_{T_1}(p)\right] &= E^I_t\left[\Delta F_{T_1}(p)\left(r^f_{R_1} - r^f_{R_2}\right)\right] = \\ &= E^I_t\left[\Delta F_{T_1}(p)\right]\left(r^f_{R_1} - r^f_{R_2}\right) = 0. \end{aligned}$$

Therefore, from the perspective of investors, the expected excess return always equals the risk premium.

Consider now how *systematic cash flow news* and *discount rate news* terms in the

return decomposition is expected to show up in the data. Because the perceived macroeconomic risk has so far exhibited mean-reversion (evident e.g. from the VIX time series) and is captured by $F_t(p)$ in the model, one would expect the discount rate and systematic cash flow news to be zero on average. In particular, mean-reversion implies that any deviations from the mean are undone in the long-run: for any positive discount rate news, for instance, the market must experience a comparable negative one eventually. Assuming no material trending changes in the corresponding weights $\rho_t^f(p)$ and $\bar{\rho}_{t,\tilde{D}}^i$, average discount rate and systematic cash flow news would thus be expected to equal zero in the data.

Consider next *idiosyncratic cash flow news*, which is driven by revisions in future idiosyncratic bad regime risk. It is well known that small firm cash flows are generally more volatile and prone to bad regimes than cash flows of large firms (see e.g. Herskovic et al. (2020)), since the latter benefit from the economies of scale and the diversification of supply networks and customer bases. Hence, one would expect idiosyncratic perceived bad regime risk to be trending downwards empirically as firms mature. In that case, *idiosyncratic cash flow news* term would be expected to be positive in the data on average for successful firms. The phenomenon could either reflect gradual resolution of uncertainty in favor of successful firm outcomes, or the market's inability to differentiate between over-achievers and under-achievers at an early stage.

Idiosyncratic cash flow news effect is very hard to generate in existing models for two reasons. Firstly, it would require ex-ante modeling of good regime beliefs as upward-trending over time. Because investors cannot presumably be a priori sure about growth dynamics of individual young firms, such design would be hard to justify. Secondly, in models in which true belief dynamics are known, investors cannot be consistently surprised. Intuitively, *idiosyncratic cash flow news* effect exists precisely because investors learn and adjust their understanding of firm dynamics over time.

Lastly, because subjective expected growth rate of idiosyncratic cash flows is a belief-weighted average of the growth expectations in the two regimes, the realized growth in the good regime will on average be above the expected growth and the opposite is true about the bad regime. Good regime cash flow levels above the subjective expectation is merely a compensation for losses that will be incurred in bad times: as long as regime beliefs are stationary and on average correct, the two effects will cancel out and *cash flow surprise* term will on average equal zero in the data. However, if a given firm experiences lower incidence of bad regimes than the market anticipated, its excess cash flows above the expectation will not be completely washed out by bad regime episodes

and the stock will confer a premium for unrealized bad regime risk.

This phenomenon cannot be observed in rational models in which subjective beliefs coincide with objective probability distributions, because subjective expectations in such models must equal average observed outcomes almost surely. In contrast, in the context of this paper, compensation for unrealized risk is expected to be positive for successful firms as long as the market cannot perfectly differentiate between over-achievers and under-achievers at an early stage of a firm lifetime. Note also that the compensation for unrealized risk is on top of the compensation due to the general possibility of bad regimes captured by the subjective risk premium.

4.3.1 Growth Component of Average Returns at the Market Level

The intuition might suggest that the growth component of returns should disappear in the aggregate: while some firms grow, others decline and the net effect should equal zero on a market level. This, however, is not true, because market-index-level outcomes are by construction tilted towards the successful firms. For instance, S&P500 firms that do well keep contributing to the index with increasing weight, while firms that do badly are simply dropped from the index and replaced with more successful peers.

4.4 Cyclicality Gap Between Subjective Risk Premia and Average Realized Excess Returns

Nagel and Xu (2023) show that subjective risk premia measured by surveys exhibit much less cyclicality than realized excess returns. In particular, the authors show that the realized excess returns over 1-year forecast horizon are much higher during periods of high macroeconomic risk than during periods of low risk. In contrast, the corresponding discrepancy between subjective risk premia is much more modest. The authors argue the finding need not indicate investor irrationality, since high returns are not predictable out-of-sample at a time a forecast is made. This suggests that the in-sample counter-cyclicality of realized excess returns reflects unpredictable uncertainty resolution ex-post rather than changing risk aversion or a predictable pattern investors could have exploited.

The framework of this paper provides the same insight. Firstly, because the subjective expected excess return always equals the risk premium and the risk premium exhibits only modest counter-cyclicality, subjective expected returns exhibit modest

counter-cyclicality. Secondly, because most of the heightened risk periods have not materialized in the past (as evidenced e.g. by time series of VIX), bad times in the data (episodes with low $F_t(p)$) predict quick recoveries (subsequent episodes with positive revisions in $F_t(p)$). Hence, in the data low $F_t(p)$ is on average associated with positive discount rate news, positive *cash flow growth surprise,* and positive *systematic cash flow news.* In contrast, because macroeconomic risk has so far exhibited mean-reversion, high $F_t(p)$ would not on average be associated with positive news or surprises. Hence, compared to the subjective risk premia, the realized excess returns over one-year forecast horizon in Nagel and Xu (2023) are much more sensitive to perceived macroeconomic risk.

The findings of Nagel and Xu (2023) and the model mechanism both indicate that belief revisions are unpredictable subjectively but follow a pattern in the data. The intuition behind this claim goes back to the main axiom of the paper: regime risk dynamics are revealed ex-post but not known ex-ante. The fact that belief revisions are predictable in the data merely reflects ex-post uncertainty resolution that has been favorable so far on average. The idea that investors do not incorporate the observed pattern of belief revisions in their expectations hinges on non-stationarity of risk dynamics: the past experiences have limited relevance to a current disturbance if the latter is driven by unique circumstances markets have little experience of.

4.5 The Expectation Hypothesis of the Term Structure of Risk Free Rates

This section shows why empirical tests reject the expectations hypothesis in the data even though it holds subjectively.

5 Conclusion

The paper introduces a new regime-switching model in which investors know true dynamics of cash flows and aggregate consumption conditional on asset-level and economy-wide expansions and contractions, but do not know true transition dynamics between regimes. It aims to formalize the idea and illustrate the implications of the notion that true regime risk dynamics are unknowable ex-ante and only revealed ex-post. All such implications could perhaps be united under a single "umbrella" claim: ex-ante unknowable regime dynamics generate patterns in the data that are not expected subjectively.

Hence, many phenomena that seem anomalous in existing models could merely be reflecting the effects of changing risk perceptions. While the paper discusses a few such anomalies, it only starts to scratch the surface.

Below, I briefly discuss five strands of potential follow-up work on top of the empirical implementation of the model and its application to various anomalies. All directions are facilitated by two novel modeling designs: the possibility of idiosyncratic bad regimes and the replacement of true regime transition dynamics with subjective regime beliefs across different horizons.

Microfounding Regime Beliefs

Although treating regime beliefs as a model primitive has its advantages, the framework could benefit from a theory of time-variation in regime beliefs. One way to accomplish this is using the Rational Inattention (RI) theory briefly discussed in Section 2.2. In a separate paper[5] that predates this work, I have incorporated Rational Inattention (RI) framework of Matejka and McKay (2015) in a simple asset pricing setting to model regime belief formation as an outcome of RI learning. The model endogenously yields a closed form expression of regime probabilities that depends on the binary signal realization s and stochastic cost of learning λ about different future time periods. A pair of s and λ span all values of regime probabilities from 0 to 1. Hence, they introduce no restrictions on possible empirical values of regime probabilities and can be backed out using VIX-based calibration methodology without affecting its feasibility.

Diagnostic Expectations and Regime Beliefs

The model might be well suited for studying the equilibrium implications of diagnostic expectations of Bordalo et al. (2019). Because the model of this paper is solved under arbitrary regime beliefs that satisfy dynamic consistency, reduced form parametrizations of regime belief updating can be directly incorporated on top of the existing framework. In this case, distortions in regime beliefs will pass down to distortions in cash flow growth expectations and the equilibrium SDF defined with respect to the objective distribution.

[5]https://www.researchgate.net/publication/385142504_Asset_Pricing_During_Potential_Disasters

Mapping Investor Surveys to Asset Pricing Models

The framework of this paper might also facilitate more effective mapping between investor surveys and asset pricing models. One challenge in mapping investor surveys to asset pricing models is that most models either assume normally distributed beliefs or depend on parameters that are not directly measurable or intuitive to the human mind. In contrast, subjective beliefs in this paper are a function of subjective regime probabilities across different horizons, and means, volatilities and correlations of consumption/dividend growth rates conditional on good and bad regimes. Because all terms are intuitive and together accommodate a wide range of beliefs, it might facilitate more precise accounting of investor beliefs. Such accounting of investor beliefs in turn provides an alternative way of calibrating the model and could be useful for separating belief and preference effects in empirical measures of time-varying risk aversion, which is in the interests of the Dutch pensions industry (Bokern et al. 2021).

Theory of Time-Varying Factor Betas

It is well-known that betas of various factors are time-varying, yet a theory of time-variation is missing in the literature. The generality of the framework might provide an useful avenue for developing such a theory. Preliminary analysis suggests that factor betas can be obtained in closed form and would depend on regime beliefs, relative valuations of systematic and idiosyncratic cash flow streams, and factor covariance with asset cash flows within different regimes. For instance, a decline in idiosyncratic bad regime likelihood of any asset would enhance the value of its idiosyncratic cash flow stream and decrease the return exposure to systematic factors. Because the model provides calibrable closed-form expressions for the values of systematic and idiosyncratic cash flow streams, it might also provide calibrable closed form expressions for forward-looking factor betas that can be directly compared against data. If successful, the theory might find useful applications in investment management.

Web3 Cryptocurrency Valuation

Many cryptocurrencies that underpin Web3 projects are much like stocks: they entitle holders to governance rights and a fraction of the Web3 network revenue. Web3 projects are in turn like firms with measurable fundamentals, because they create products and services that aim to disrupt or improve already well-established industries. However,

estimating discount rates and expected cash flows of the Web3 projects directly is challenging, while models with pre-specified cash flow law of motion must embed strong assumptions about the long-term potential of individual Web3 projects and their cash flow dynamics. The framework of this paper might provide an appealing alternative, since it accommodates different cash flow regimes without the need to specify regime transition dynamics and their relative frequencies. Instead the focus could be shifted towards estimating regime probabilities for different horizons from a normative perspective. The framework of this paper could be useful as a descriptive tool as well, since VIX-like implied volatility measures are becoming increasingly available for a range of cryptocurrencies.

6 References

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Mathematical Appendix

Provided upon request.